MATH 267 CHAPTER 17 MULTIPLE INTEGRALS

17.2 AREA AND VOLUME

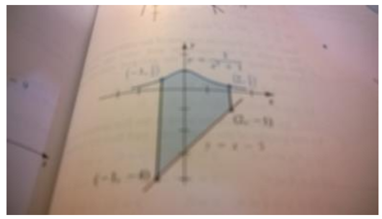
Volume $V=\int_a^b\int_{g_1(x)}^{g_2(x)} f(x,y)\,dydx$

Area $A=\iint_R dA=\int_a^b\int_{g_1(x)}^{g_2(x)} dydx=\int_a^b\left[g_2(x)-g_1(x)\right]dx$

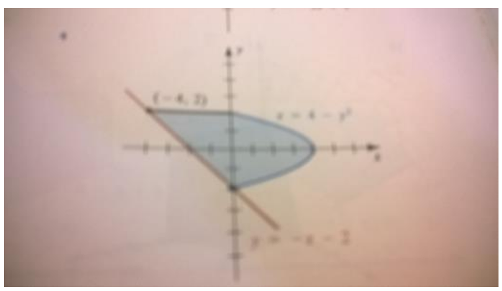
Exercises

2) Solution:

$$R_x:\quad A=\int_{-1}^{2}\int_{x-3}^{\frac{1}{x^2+1}} dy\ dx$$



4) Solution:

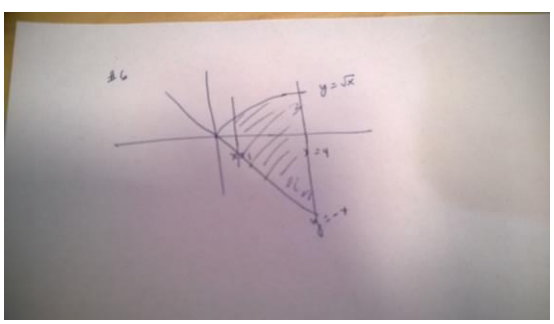


$$R_x:\quad A=\int_{-4}^{0}\int_{-x-2}^{2} dy\ dx \quad+\quad \int_{-2}^{2}\int_{0}^{4-y^2} dx\ dy$$

$$R_y:\quad A=\int_{-2}^{2}\int_{-y-2}^{4-y^2} dx\ dy$$

6) $y=\sqrt{x},\ \ y=-x,\ \ x=1,\ \ x=4$

Solution:

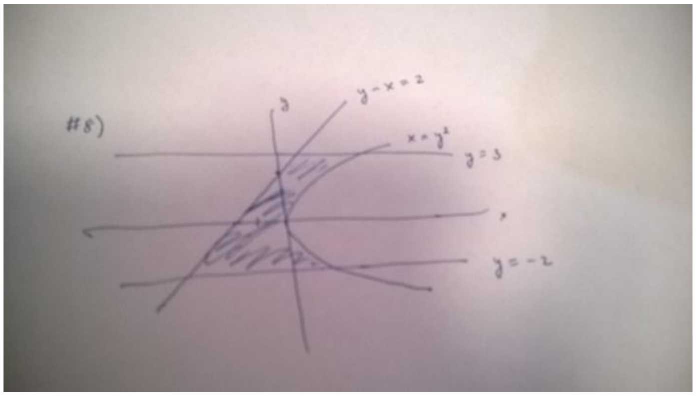


$$R_x:\quad A=\int_1^4\int_{-x}^{\sqrt{x}} dy\ dx=\int_1^4 y\Big]_{-x}^{\sqrt{x}} dx=\int_1^4\left(\sqrt{x}+x\right)dx$$

$$=\left[\frac{2}{3}x^{3/2}+\frac{x^2}{2}\right]_1^4=\frac{2}{3}(4)^{3/2}+\frac{4^2}{2}-\left(\frac{2}{3}+\frac{1}{2}\right)=\frac{16}{3}+8-\frac{2}{3}-\frac{1}{2}=\frac{73}{6}$$

8) $x = y^2,\ \ y - x = 2,\ \ y = -2,\ \ y = 3$

Solution:

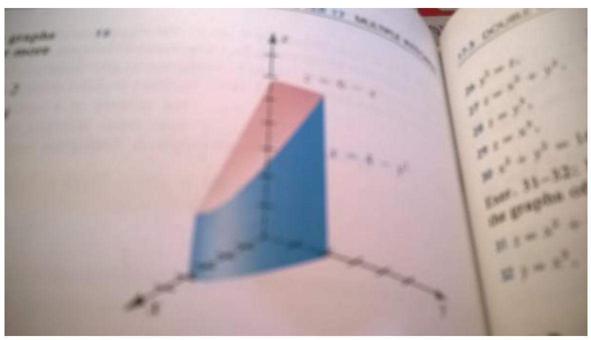


$$R_y:\ \int_{-2}^{3}\int_{y-2}^{y^2} dxdy = \int_{-2}^{3}\left[y^2-(y-2)\right]dy = \left[\frac{y^3}{3}-\frac{y^2}{2}+2y\right]_{-2}^{3}$$

$$= \frac{27}{3}-\frac{9}{2}+6-\left(\frac{-8}{3}-\frac{4}{2}-4\right)=\frac{115}{6}$$

14)

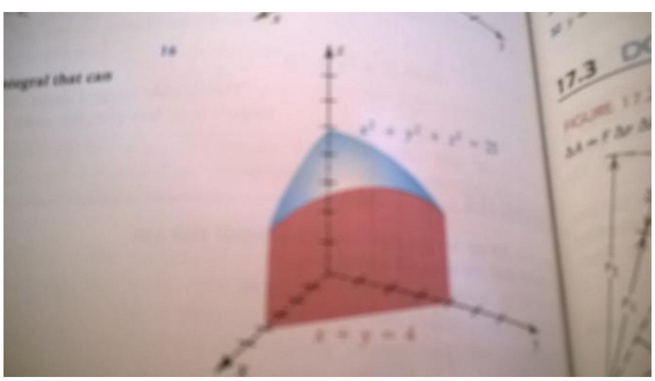
Solution:



$$V = \int_0^2\int_0^{4-x}\left(4-x^2\right)dydx$$

16)

Solution:



$$V = \int_0^4\int_0^{4-x}\sqrt{25-x^2-y^2}\ dydx$$

OR

$$V = \int_0^4\int_0^{4-y}\sqrt{25-x^2-y^2}\ dxdy$$

The following iterated double integrals represent the volume of a solid under a surface *S* and over a region *R* in the *xy*-plane. Describe *S* and sketch *R*.

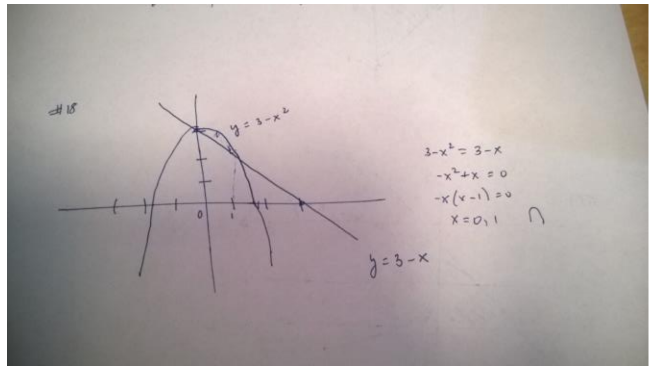
18) $\int_0^1 \int_{3-x}^{3-x^2} \sqrt{25-x^2-y^2}\ dydx$

Solution:

$$z=\sqrt{25-x^2-y^2}$$
$$z^2=25-x^2-y^2$$
$$x^2+y^2+z^2=25$$

S is the top half of the sphere $x^2+y^2+z^2=25$.

R is the region bounded by the parabola $y=3-x^2$ and the line $y=3-x$.



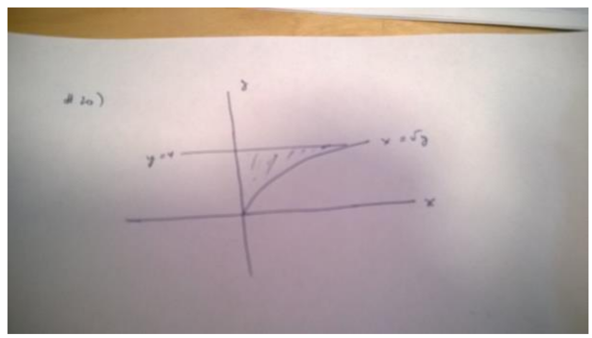
20) $\int_0^4 \int_0^{\sqrt{y}} \sqrt{x^2+y^2}\ dxdy$

Solution:

$$z=\sqrt{x^2+y^2}$$
$$z^2=x^2+y^2$$
$$x^2+y^2-z^2=0$$

S is the top half of the cone.

R is the region bounded by $x=\sqrt{y}$, the $y-$axis, and $y=4$

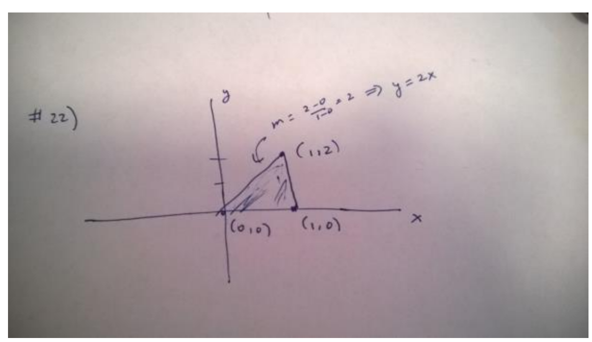


Find the volume of the solid that lies under the graph of the equation and over the region in the *xy*-plane bounded by the polygon with the given vertices.

22) $z=x^2+4y^2;\ (0,0),\ (1,0),\ (1,2)$

Solution:

$$V=\int_0^1\int_0^{2x}\left(x^2+4y^2\right)dydx$$
$$=\int_0^1\left[x^2y+\frac{4y^3}{3}\right]_0^{2x}dx$$
$$=\int_0^1\left(2x^3+\frac{32}{3}x^3\right)dx$$
$$=\int_0^1\frac{38}{3}x^3dx$$
$$=\frac{38}{3}\frac{x^4}{4}\Big]_0^1=\frac{38}{12}=\frac{19}{6}$$

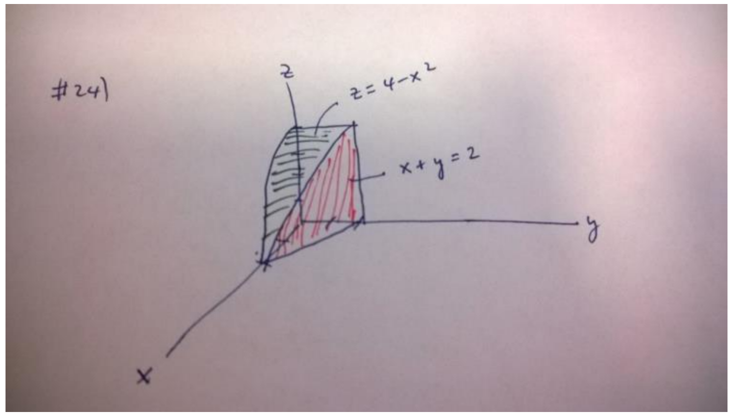


Sketch the solid in the first octant bounded by the graphs of the equations and find its volume.

24) $z = 4 - x^2,\ \ x + y = 2,\ \ x = 0,\ \ y = 0,\ \ z = 0$

Solution:

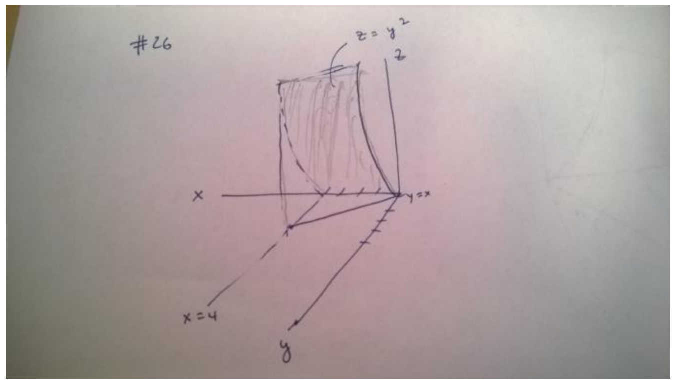
$$\begin{aligned}
V &= \int_0^2 \int_0^{2-x} \left(4 - x^2\right) dy\,dx \\
&= \int_0^2 \Big[4y - x^2 y\Big]_0^{2-x} dx \\
&= \int_0^2 \Big[4(2-x) - x^2(2-x)\Big] dx \\
&= \int_0^2 \Big[8 - 4x - 2x^2 + x^3\Big] dx \\
&= \left[8x - 2x^2 - \frac{2}{3}x^3 + \frac{1}{4}x^4\right]_0^2 \\
&= 16 - 8 - \frac{16}{3} + 4 = \frac{20}{3}
\end{aligned}$$



26) $y^2 = z,\ \ y = x,\ \ x = 4,\ \ z = 0$

Solution:

$$\begin{aligned}
V &= \int_0^4 \int_0^x y^2\,dy\,dx \\
&= \int_0^4 \left.\frac{y^3}{3}\right]_0^x dx \\
&= \int_0^4 \frac{x^3}{3} dx \\
&= \left.\frac{1}{12}x^4\right]_0^4 = \frac{64}{3}
\end{aligned}$$

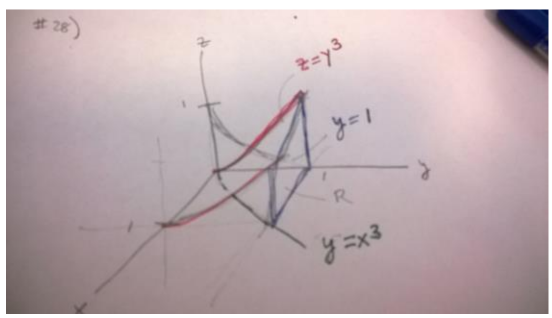


28)

$z = y^3,\ \ y = x^3,\ \ x = 0,\ \ z = 0,\ \ y = 1$

Solution:

$$V = \int_0^1 \int_{x^3}^1 y^3\,dy\,dx = \frac{3}{13}$$



30) $x^2 + y^2 = 16,\ \ x = z,\ \ y = 0,\ \ z = 0$

Solution:

$$V = \int_0^4 \int_0^{\sqrt{16-x^2}} x\,dy\,dx = \frac{64}{3}$$