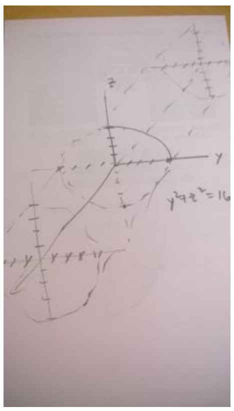
MATH 267 CHAPTER 14 VECTORS AND SURFACES

14.6 SURFACES

Definition: Let C be a curve in a plane, and let l be a line that is not in a parallel plane. The set of all lines that are parallel to l and intersect C is a **cylinder**. The curve C in the plane is called a **directrix** for the cylinder, and each line through C parallel to l is a **ruling** of the cylinder. A **right circular cylinder** is obtained if C is a circle in a plane and l is a line perpendicular to the plane.

Exercise
2) $y^2 + z^2 = 16$



In two dimensions, the graph of any second-degree equation in x and y,

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

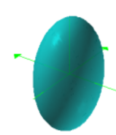
is a **conic section** (except for degenerate cases). In three dimensions, the graph of a second-degree equation in x, y, and z,

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0,$$

is a **quadric surface** (except for degenerate cases). We limit the discussion to the case where $D, E, F, G, H,$ and I are all zero.

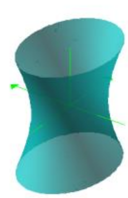
There are 3 types of quadric surfaces: *ellipsoids*, *hyperboloids*, and *paraboloids*.

Ellipsoid $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

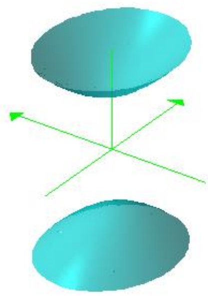


Hyperboloid of one sheet $$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

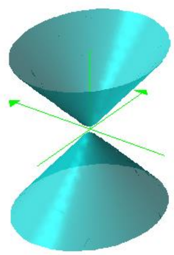
axis is z – axis



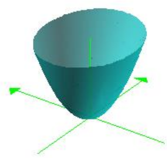
Hyperboloid of two sheets $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ axis is z – axis



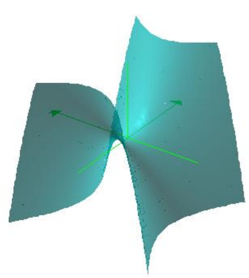
Cone: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$

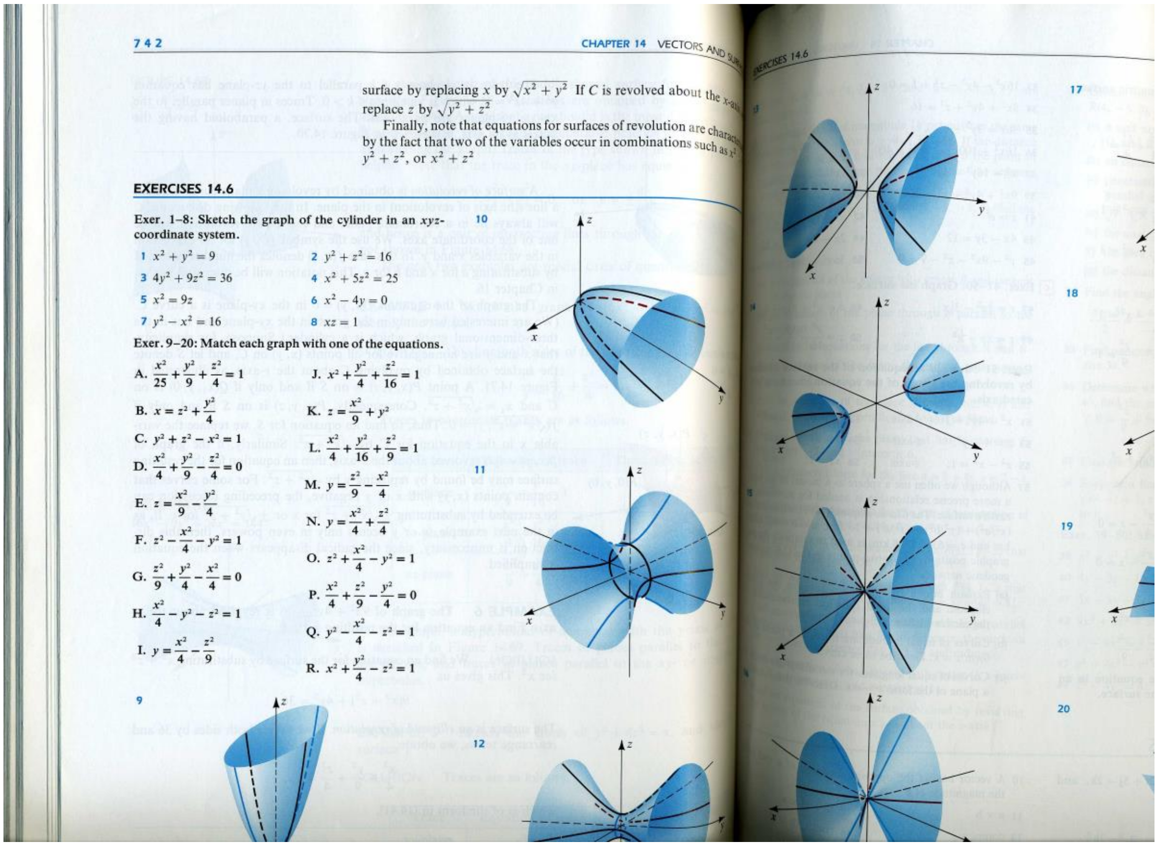


Paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=cz$ axis is z – axis



Hyperbolic Paraboloid $\frac{y^2}{a^2}-\frac{x^2}{b^2}=cz$ axis is z – axis





surface by replacing x by $\sqrt{x^2 + y^2}$ If C is revolved about the x-axis, replace z by $\sqrt{y^2 + z^2}$

Finally, note that equations for surfaces of revolution are characterized by the fact that two of the variables occur in combinations such as $x^2 + y^2$, $y^2 + z^2$, or $x^2 + z^2$

EXERCISES 14.6

Exer. 1–8: Sketch the graph of the cylinder in an xyz-coordinate system.

1 $x^2 + y^2 = 9$ 2 $y^2 + z^2 = 16$

3 $4y^2 + 9z^2 = 36$ 4 $x^2 + 5z^2 = 25$

5 $x^2 = 9z$ 6 $x^2 - 4y = 0$

7 $y^2 - x^2 = 16$ 8 $xz = 1$

Exer. 9–20: Match each graph with one of the equations.

A. $\frac{x^2}{25} + \frac{y^2}{9} + \frac{z^2}{4} = 1$ J. $x^2 + \frac{y^2}{4} + \frac{z^2}{16} = 1$

B. $x = z^2 + \frac{y^2}{4}$ K. $z = \frac{x^2}{9} + y^2$

C. $y^2 + z^2 - x^2 = 1$ L. $\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{9} = 1$

D. $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{4} = 0$ M. $y = \frac{z^2}{9} - \frac{x^2}{4}$

E. $z = \frac{x^2}{9} - \frac{y^2}{4}$ N. $y = \frac{x^2}{4} + \frac{z^2}{4}$

F. $z^2 - \frac{x^2}{4} - y^2 = 1$ O. $z^2 + \frac{x^2}{4} - y^2 = 1$

G. $\frac{z^2}{9} + \frac{y^2}{4} - \frac{x^2}{4} = 0$ P. $\frac{x^2}{4} + \frac{z^2}{9} - \frac{y^2}{4} = 0$

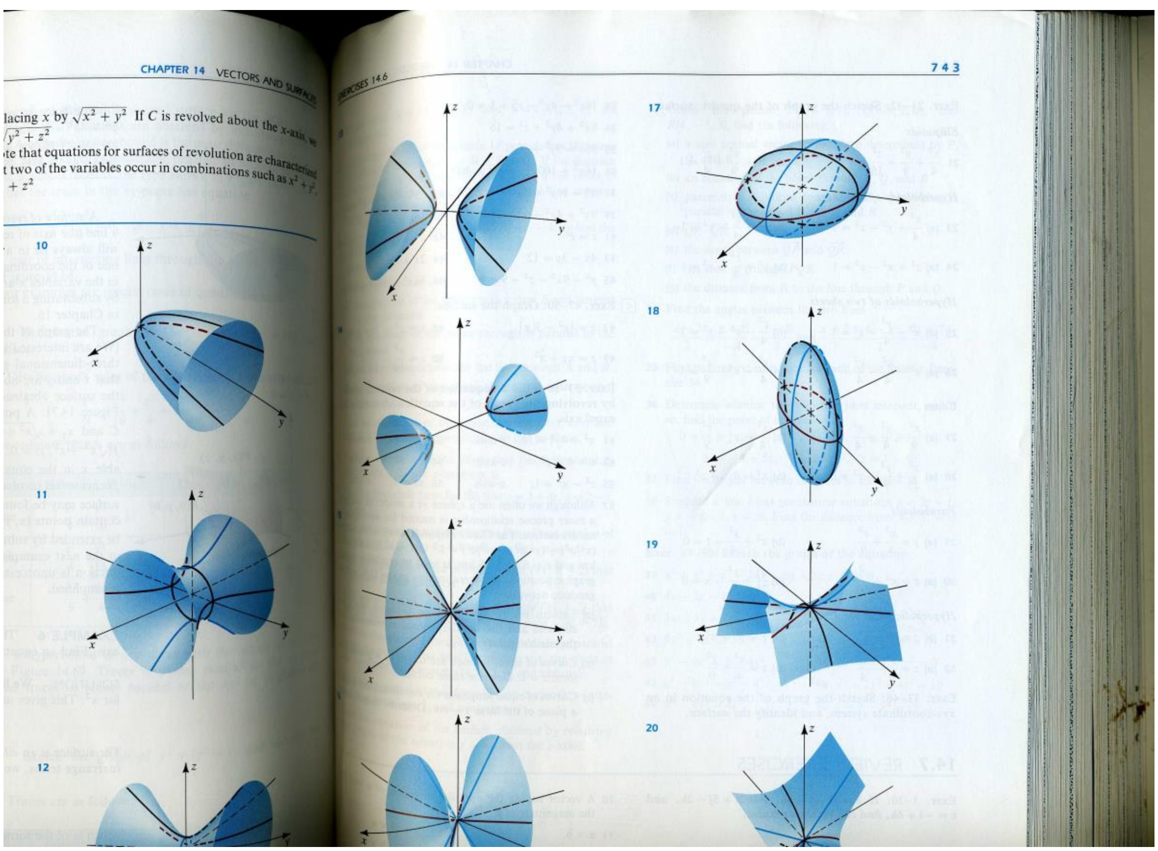
H. $\frac{x^2}{4} - y^2 - z^2 = 1$ Q. $y^2 - \frac{x^2}{4} - z^2 = 1$

I. $y = \frac{x^2}{4} - \frac{z^2}{9}$ R. $x^2 + \frac{y^2}{4} - z^2 = 1$

9 10 11 12

13 14 15 16

17 18 19 20



CHAPTER 14 VECTORS AND SURFACES

9 10 11 12

13 14 15 16

17 18 19 20

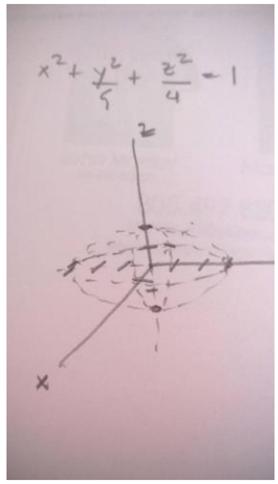
22) $x^2+\frac{y^2}{9}+\frac{z^2}{4}=1$ **ellipsoid**

Traces:

xy-plane: **ellipse** $x^2+\frac{y^2}{9}=1$

yz-plane: **ellipse** $\frac{y^2}{9}+\frac{z^2}{4}=1$

xz-plane: **ellipse** $x^2+\frac{z^2}{4}=1$



24) a) $z^2+x^2-y^2=1$ **hyperboloid of one sheet, axis *y*-axis**

Traces:

xy-plane: **hyperbola** $x^2-y^2=1$

yz-plane: **hyperbola** $z^2-y^2=1$

xz-plane: **circle** $z^2+x^2=1$

26) a) $z^2-\frac{x^2}{4}-\frac{y^2}{4}=1$ **hyperboloid of two sheets, axis *z*-axis**

Traces:

xy-plane: **none** $-\frac{x^2}{4}-\frac{y^2}{4}=1$

yz-plane: **hyperbola** $z^2-\frac{y^2}{4}=1$

xz-plane: **hyperbola** $z^2-\frac{x^2}{4}=1$

28) a) $\frac{x^2}{25}+\frac{y^2}{9}-z^2=0$ **cone, axis *z*-axis**

Traces:

xy-plane: **point (0,0,0)** $\frac{x^2}{25}+\frac{y^2}{9}=0$

yz-plane: **two lines** $\frac{y^2}{9}-z^2=0 \Rightarrow z=\pm\frac{y}{3}$

xz-plane: **two lines** $\frac{x^2}{25}-z^2=0 \Rightarrow z=\pm\frac{x}{5}$

30) a) $z=x^2+\frac{y^2}{9}$ **paraboloid**

Traces:

xy-plane: **point (0,0,0)** $0=x^2+\frac{y^2}{9}$

yz-plane: **parabola** $z=\frac{y^2}{9}$

xz-plane: **parabola** $z=x^2$

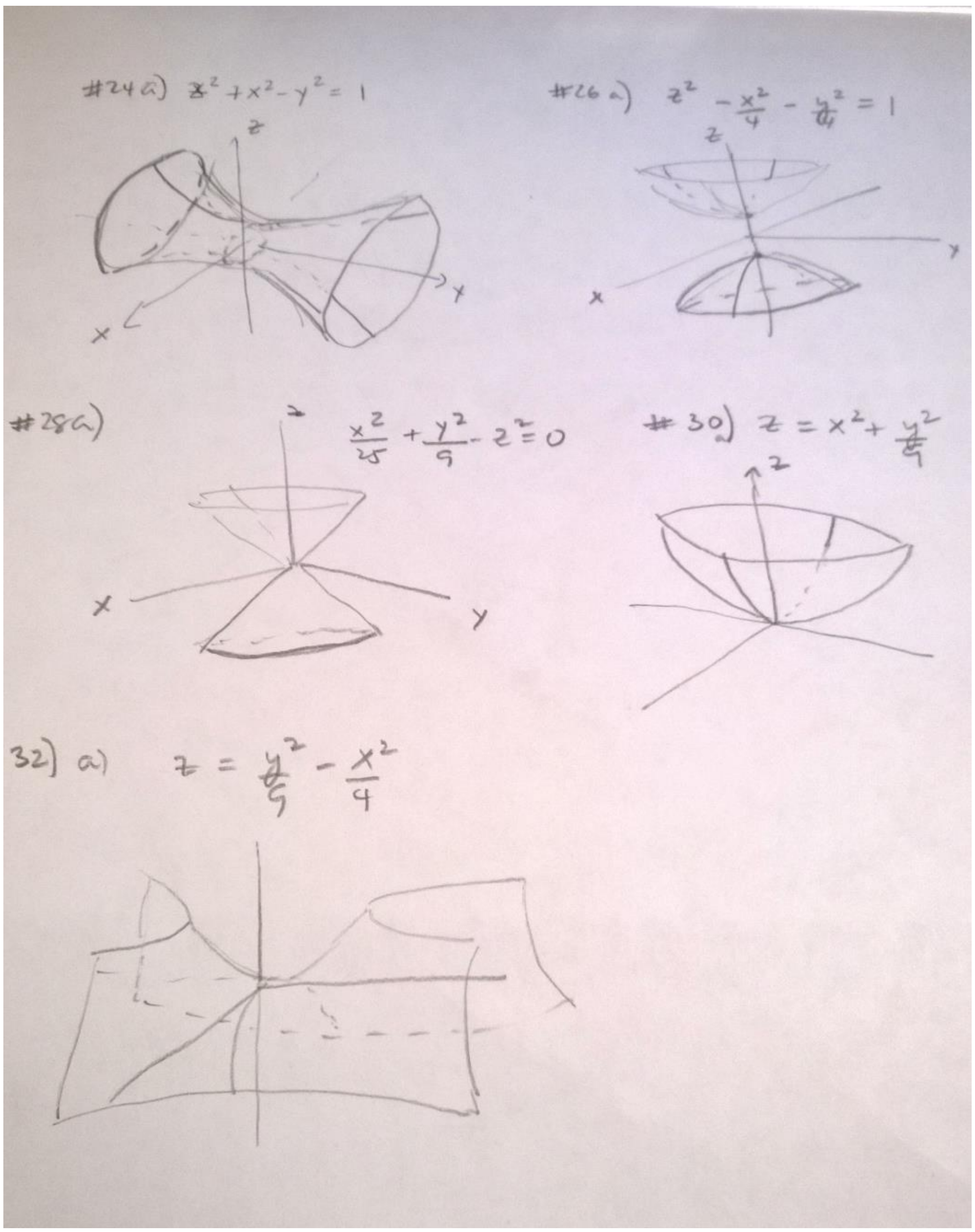
32) a) $z = \frac{y^2}{9} - \frac{x^2}{4}$ **hyperbolic paraboloid**

Traces:

xy-plane: **two lines** $0 = \frac{y^2}{9} - \frac{x^2}{4} \Rightarrow y = \pm\frac{3}{2}x$

yz-plane: **parabola** $z = \frac{y^2}{9}$

xz-plane: **parabola** $z = -\frac{x^2}{4}$



A **surface of revolution** is obtained by revolving a plane curve *C* about a line (the axis of revolution) in the plane.

We let *C* lie in a coordinate plane and the axis of revolution will be one of the coordinate axes. We will use $f(x,y)$ for an expression in the variables *x* and *y*.

The graph of $f(x,y)=0$ in the *xy*-plane is a curve *C*.

We will assume that *x* and *y* are nonnegative for all points (x,y) on *C*.

Let *S* denote the surface obtained by revolving *C* about the *y*-axis.

A point $P(x,y,z)$ is on *S* $\Leftrightarrow Q(x_1,y,0)$ is on *C* and $x_1=\sqrt{x^2+z^2}$

$$\Leftrightarrow f\left(\sqrt{x^2+z^2},y\right)=0$$

Thus, to find an equation for *S*, we replace the variable *x* in the equation for *C* by $\sqrt{x^2+z^2}$

Similarly, if the graph of $f(x,y)=0$ is revolved about the *x*-axis, we replace *y* by $\sqrt{y^2+z^2}$.

For some curves that contain points (x,y) with *x* or *y* negative, replace *x* by $\pm\sqrt{x^2+z^2}$ or *y* by $\pm\sqrt{y^2+z^2}$.

Exercise

52) $y^2=4x;\ \ x-\text{axis}$

Solution: We replace *y* by $\sqrt{y^2+z^2}$ to get $y^2+z^2=4x$ which is a paraboloid.