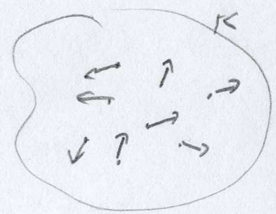
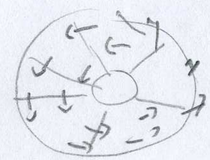
18.1 Vector Fields



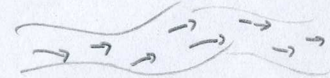
vector field

ex:

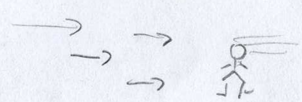


rotating wheel

① velocity field



flow of water



wind

② force field

vector field in 3D:

$\vec{F} : \mathbb{R}^3 \to V_3 \; (\subseteq)$

$\vec{F}(x,y,z) = M(x,y,z)\,\vec{i} + N(x,y,z)\,\vec{j} + P(x,y,z)\,\vec{k}$

M, N, P scalar functions

vector field in 2D

$\vec{F} : \mathbb{R}^2 \to V_2 \; (\subseteq)$

$\vec{F}(x,y) = M(x,y)\,\vec{i} + N(x,y)\,\vec{j}$

Ex#2 $\vec{F}(x,y) = -x\vec{i} + y\vec{j}$

$\vec{F}(1,2) = -\vec{i} + 2\vec{j}$

$\vec{F}(-1,1) = \vec{i} + \vec{j}$

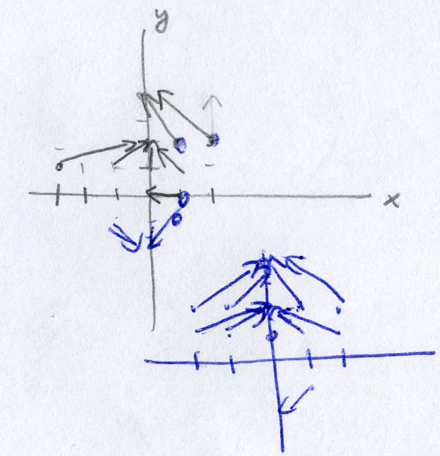
$F(0,1) = 0\vec{i} + \vec{j}$

$F(1,0) = -\vec{i} + 0\vec{j}$

$F(1,1) = -\vec{i} + \vec{j}$

$F(2,2) = -2\vec{i} + 2\vec{j}$

$F(-3,1) = 3\vec{i} + \vec{j}$



Def Let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ be the position vector for (x,y,z)

$\vec{u} = \dfrac{\vec{r}}{\|\vec{r}\|}$

A vector field $\vec{F}$ is an inverse square field if

$$\vec{F}(x,y,z) = \frac{c}{\|\vec{r}\|^2}\vec{u} = \frac{c}{\|\vec{r}\|^3}\vec{r},$$

where c is a scalar.

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$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$$\vec{F}(x,y,z) = \frac{c}{\|\vec{r}\|^3}\vec{r} = \frac{c}{(x^2+y^2+z)^{3/2}}(x\vec{i} + y\vec{j} + z\vec{k})$$

If $c < 0$, $\vec{F}(x,y,z)$ is in opposite direction as $\vec{r}$

$\Rightarrow$ towards O

If $c > 0$, $\vec{F}(x,y,z)$ is in same direction as $\vec{r}$

$\Rightarrow$ away from O