Elementary Algebra Skill-Builder # E – 8 Applying Combined Exponent Rules

We have the following exponent rules for any nonzero real numbers a and b, and positive integers m and n,

Product Rule: $a^m \cdot a^n = a^{m+n}$

Power Rule: $(a^m)^n = a^{mn}$

Distributivity of Exponentiation over Multiplication: $(ab)^n = a^n b^n$

Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$

Zero Exponent Rule: $a^0 = 1$

Negative Exponent Rule: $a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$

Distributivity of Exponentiation over Division: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Examples

1. Simplify:
$$\left(\frac{2a^{-1}b}{5a^2b^0c^{-1}}\right)^2$$

Solution: We could simplify the expression inside the quantity first or we can distribute the exponent 2 first using the two distributive properties. Let us do the latter to get

$$\left(\frac{2a^{-1}b}{5a^2b^0c^{-1}}\right)^2 = \frac{2^2a^{-2}b^2}{5^2a^4b^0c^{-2}}.$$

Next, we can use the zero and negative exponent rules as well as evaluate 2^2 and 5^2 to get $\frac{4b^2\mathbf{c}^2}{25a^4\mathbf{a}^2}$.

Then, we can apply the product rule to get the final answer of $\frac{4b^2c^2}{25a^6}$.

2. Simplify:
$$\frac{3x^3 \left(-2x^{-1}y^4\right)^{-3}}{\left(-3x^2y^{-4}\right)^2}$$

Solution: We first apply the distributivity of exponentiation over multiplication to get

$$\frac{3x^3\left(-2x^{-1}y^4\right)^{-3}}{\left(-3x^2y^{-4}\right)^2} = \frac{3x^3\left(-2\right)^{-3}x^3y^{-12}}{\left(-3\right)^2x^4y^{-8}}.$$

Then apply the negative exponent rule to get $\frac{3x^3x^3\mathbf{y}^8}{(-3)^2(-2)^3x^4\mathbf{y}^{12}}$. Now apply the product and quotient

rules and evaluate the numeric expressions to get $\frac{3x^6y^{-4}}{-72x^4}$ and further simplify to get $-\frac{x^2}{24y^4}$.

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Simplify the following using the exponent rules.

1.
$$\frac{4^0 x^{-3} y^5}{4 x y^{-1}}$$

2.
$$(3^{-1}a^0b^5c^{-3})^{-4}$$

$$3. \quad \left(\frac{2x^3y^{-2}}{3^0x^{-3}y^{-1}}\right)^{-3}$$

4.
$$\frac{(5y)^{-1}(-3y^{-2})^{-1}}{5y^{0}(3y)^{-2}}$$

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Answer Key:

- 1. $\frac{y^6}{4x^4}$
- 2. $\frac{81c^{12}}{b^{20}}$
- 3. $\frac{y^3}{8x^{18}}$
- **4.** $-\frac{3y^3}{25}$

Prepared by: Teresa V. Sutcliffe Summer 2012