

Elementary Algebra
Skill-Builder # SQRT – 1B
Simplifying Square Root Radicals: Radicand Consists of Variables

We now consider the case when the radicand is a power of a variable or a product of powers of variables. To simplify the discussion, we will assume that all variables represent positive real numbers. For example if we want to find $\sqrt{x^2}$, we saw in the case of positive constants that when we have the exponential form, we can “divide” the exponent by the index giving us $\sqrt{x^2} = \sqrt[2]{x^2} = x$. Of course, the mathematical explanation is x is the positive number that when squared gives x^2 .

Examples Find the following.

1. $\sqrt{a^6}$

Solution: We can again “divide” as follows: $\sqrt{a^6} = \sqrt[2]{a^6} = a^3$. Arguing mathematically, we have the square of a^3 which is $(a^3)^2$ is a^6 and thus the principal square root of a^6 is a^3 .

2. $\sqrt{x^4 y^{10}}$

Solution: Following the “division” argument we get $\sqrt{x^4 y^{10}} = \sqrt[2]{x^4 y^{10}} = x^2 y^5$. Of course the correct mathematical reasoning is $\sqrt{x^4 y^{10}} = x^2 y^5$ since $(x^2 y^5)^2 = x^4 y^{10}$.

3. $\sqrt{\frac{a^8}{b^2 c^4}}$

Solution: Verify using either argument that the answer is $\frac{a^4}{bc^2}$.

Now, let’s consider the case when the exponent is odd, i.e. when it is not divisible by 2.

4. $\sqrt{n^3}$

Solution: We apply the same concept we used for numbers, i.e. we write the radicand as a product of a perfect square and something that is not necessarily a perfect square. We see that

$$\sqrt{n^3} = \sqrt{n^2 \cdot n} = \sqrt{n^2} \cdot \sqrt{n} = n\sqrt{n}.$$

Let’s do this for a product of powers of several variables.

5. $\sqrt{x^5 y^{11}}$

Solution: We can write $\sqrt{x^5 y^{11}}$ as $\sqrt{x^4 \cdot x \cdot y^{10} \cdot y}$. We can take out the square roots of x^4 and y^{10} and these are x^2 and y^5 , respectively. Thus, we get $x^2 y^5 \sqrt{xy}$ for the final answer.

We can also do a similar process for the quotient of powers of several variables.

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Find the following. Assume all variables represent positive real numbers.

1. $\sqrt{a^8}$	2. $\sqrt{c^{24}}$
3. $\sqrt{\frac{x^4}{y^{20}}}$	4. $\sqrt{p^{10}r^{32}}$
5. $\sqrt{y^9}$	6. $\sqrt{w^{25}}$
7. $\sqrt{n^3y^7}$	8. $\sqrt{\frac{a^5b^9}{c^4d^2}}$

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Answers

1. a^4	2. $\frac{2}{7}$
3. $\frac{x^2}{y^{10}}$	4. p^5r^{16}
5. $y^4\sqrt{y}$	6. $w^{12}\sqrt{w}$
7. $ny^3\sqrt{ny}$	8. $\frac{a^2b^4\sqrt{ab}}{c^2d}$

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