

**Elementary Algebra**  
**Skill-BUILDER # SQRT – 5B**  
**Rationalizing Denominators: Two-Term Radical in the Denominator**

Now let us consider the case when the denominator consists of two terms where one or both of the terms are not rational. For this case, we will use the special product

$$(a + b)(a - b) = a^2 - b^2.$$

Let us see how this will work in the following examples.

**Examples** Rationalize the denominator.

1.  $\frac{3}{\sqrt{5} + \sqrt{6}}$

Solution: Note that the denominator looks like a sum of two terms  $a + b$ . If we want to use the special product formula given above, then we need to multiply this denominator by the difference of the two terms  $a - b$  which in this case is  $\sqrt{5} - \sqrt{6}$ . We must of course remember to multiply the numerator by the same expression. We then get

$$\frac{3}{\sqrt{5} + \sqrt{6}} = \frac{3}{\sqrt{5} + \sqrt{6}} \cdot \frac{\sqrt{5} - \sqrt{6}}{\sqrt{5} - \sqrt{6}} = \frac{3(\sqrt{5} - \sqrt{6})}{(\sqrt{5})^2 - (\sqrt{6})^2} = \frac{3(\sqrt{5} - \sqrt{6})}{5 - 6} = -3(\sqrt{5} - \sqrt{6}) \text{ or } -3\sqrt{5} + 3\sqrt{6}.$$

2.  $\frac{2}{\sqrt{7} - 3}$

Solution:

$$\frac{2}{\sqrt{7} - 3} = \frac{2}{\sqrt{7} - 3} \cdot \frac{\sqrt{7} + 3}{\sqrt{7} + 3} = \frac{2(\sqrt{7} + 3)}{(\sqrt{7})^2 - 3^2} = \frac{2(\sqrt{7} + 3)}{7 - 9} = \frac{\cancel{2}(\sqrt{7} + 3)}{-\cancel{2}} = -(\sqrt{7} + 3) \text{ or } -\sqrt{7} - 3$$

3.  $\frac{3\sqrt{2}}{4\sqrt{3} + 3\sqrt{5}}$

Solution:

$$\begin{aligned} \frac{3\sqrt{2}}{4\sqrt{3} + 3\sqrt{5}} &= \frac{3\sqrt{2}}{4\sqrt{3} + 3\sqrt{5}} \cdot \frac{4\sqrt{3} - 3\sqrt{5}}{4\sqrt{3} - 3\sqrt{5}} = \frac{3\sqrt{2}(4\sqrt{3} - 3\sqrt{5})}{(4\sqrt{3})^2 - (3\sqrt{5})^2} = \frac{3\sqrt{2}(4\sqrt{3} - 3\sqrt{5})}{16 \cdot 3 - 9 \cdot 5} = \frac{3\sqrt{2}(4\sqrt{3} - 3\sqrt{5})}{48 - 45} \\ &= \frac{\cancel{3}\sqrt{2}(4\sqrt{3} - 3\sqrt{5})}{\cancel{3}} = \sqrt{2}(4\sqrt{3} - 3\sqrt{5}) = 4\sqrt{6} - 3\sqrt{10} \end{aligned}$$

4.  $\frac{2\sqrt{2x}}{3\sqrt{x} - x\sqrt{3}}$

Solution:

$$\begin{aligned} \frac{2\sqrt{2x}}{3\sqrt{x} - x\sqrt{3}} &= \frac{2\sqrt{2x}}{3\sqrt{x} - x\sqrt{3}} \cdot \frac{3\sqrt{x} + x\sqrt{3}}{3\sqrt{x} + x\sqrt{3}} = \frac{2\sqrt{2x}(3\sqrt{x} + x\sqrt{3})}{(3\sqrt{x})^2 - (x\sqrt{3})^2} = \frac{2\sqrt{2x}(3\sqrt{x} + x\sqrt{3})}{9x - 3x^2} \\ &= \frac{6\sqrt{2x^2} + 2x\sqrt{6x}}{9x - 3x^2} = \frac{6x\sqrt{2} + 2x\sqrt{6x}}{9x - 3x^2} \end{aligned}$$

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Rationalize the denominator. Assume all variables represent positive real numbers.

1. $\frac{5}{\sqrt{3} + \sqrt{2}}$	2. $\frac{2}{\sqrt{8} - \sqrt{10}}$
3. $\frac{3}{4 - \sqrt{5}}$	4. $\frac{6}{\sqrt{12} + 2}$
5. $\frac{5\sqrt{5}}{2\sqrt{7} + 3\sqrt{2}}$	6. $\frac{\sqrt{2} + \sqrt{6}}{\sqrt{3} - \sqrt{5}}$
7. $\frac{9a\sqrt{2a}}{2a - 3\sqrt{a}}$	8. $\frac{2\sqrt{x} - y}{2\sqrt{x} + y}$

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**Answers**

1. $5\sqrt{3} + 5\sqrt{2}$	2. $-2\sqrt{2} + \sqrt{10}$
3. $\frac{4 + \sqrt{5}}{3}$	4. $\frac{3\sqrt{3} - 3}{2}$
5. $\frac{2\sqrt{35} - 3\sqrt{10}}{2}$	6. $\frac{\sqrt{6} + \sqrt{10} + 3\sqrt{2} + \sqrt{30}}{-2}$
7. $\frac{18a\sqrt{2a} + 27a\sqrt{2}}{4a - 9}$	8. $\frac{4x - 4y\sqrt{x} + y^2}{4x - y^2}$

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