**Elementary Algebra**
**Skill-Builder # SQRT – 5A**
**Rationalizing Denominators: Single-Term Radical in the Denominator**

A fraction or rational expression is usually not considered in simplest form when there are radicals in the denominator. Radicals such as $\sqrt{2}$, $\sqrt{3}$, etc. are irrational numbers and we don't usually want to see them in the denominator. The process of removing them from the denominator is called **rationalizing the denominator** which means we want to make the denominator a rational number. How do we accomplish this when there is a single square root expression in the denominator? Note that when we multiply, say, $\sqrt{2}$ by itself that we get $\sqrt{2}\cdot\sqrt{2}=\sqrt{4}$ or $\sqrt{2\cdot 2}=2$ and the square root symbol is gone! So we will do this for the following, remembering, of course, that what we multiply to the denominator should also be multiplied to the numerator to keep the value of the original expression.

**Examples** Rationalize the denominator.

**1.** $\dfrac{3}{\sqrt{5}}$

Solution: $\dfrac{3}{\sqrt{5}}=\dfrac{3}{\sqrt{5}}\cdot\dfrac{\sqrt{\mathbf{5}}}{\sqrt{\mathbf{5}}}=\dfrac{3\sqrt{5}}{\sqrt{5\cdot 5}}=\dfrac{3\sqrt{5}}{5}$

.

We can do the same thing for variables which we assume to represent positive real numbers.

**2.** $\dfrac{2a}{\sqrt{a}}$

Solution: $\dfrac{2a}{\sqrt{a}}=\dfrac{2a}{\sqrt{a}}\cdot\dfrac{\sqrt{\mathbf{a}}}{\sqrt{\mathbf{a}}}=\dfrac{2a\sqrt{a}}{\sqrt{a^2}}=\dfrac{2a\sqrt{a}}{a}=2\sqrt{a}$

We can have a factor sitting outside the radical in the denominator.

**3.** $\dfrac{3x}{4\sqrt{10}}$

Solution: In this case, there is no need to multiply by the factor 4 since this factor is radical-free.

$$\frac{3x}{4\sqrt{10}}=\frac{3x}{4\sqrt{10}}\cdot\frac{\sqrt{\mathbf{10}}}{\sqrt{\mathbf{10}}}=\frac{3x\sqrt{10}}{4\sqrt{100}}=\frac{3x\sqrt{10}}{4\cdot 10}=\frac{3x\sqrt{10}}{40}$$

Convince yourself that there is really no need to multiply by 4! ☺

There could be more than one way to rationalize the denominator.

**4.** $\dfrac{3\sqrt{2}}{5\sqrt{27}}$

Solution: We can proceed as usual, i.e. we can multiply numerator and denominator by $\sqrt{27}$ to get

$$\frac{3\sqrt{2}}{5\sqrt{27}}=\frac{3\sqrt{2}}{5\sqrt{27}}\cdot\frac{\sqrt{\mathbf{27}}}{\sqrt{\mathbf{27}}}=\frac{3\sqrt{2\cdot 27}}{5\cdot 27}=\frac{\cancel{3}\sqrt{2\cdot\boxed{3\cdot 3}\cdot 3}}{5\cdot\cancel{3}\cdot 3\cdot 3}=\frac{\cancel{3}\sqrt{6}}{5\cdot\cancel{3}\cdot 3}=\frac{\sqrt{6}}{15};$$

or, we can simplify the expression first before rationalizing the denominator as follows:

$$\frac{3\sqrt{2}}{5\sqrt{27}}=\frac{3\sqrt{2}}{5\sqrt{\boxed{3\cdot 3}\cdot 3}}=\frac{\cancel{3}\sqrt{2}}{5\cdot\cancel{3}\sqrt{3}}=\frac{\sqrt{2}}{5\sqrt{3}}\cdot\frac{\sqrt{\mathbf{3}}}{\sqrt{\mathbf{3}}}=\frac{\sqrt{6}}{5\cdot 3}=\frac{\sqrt{6}}{15}.$$

**Elementary Algebra**
**Skill-Builder # SQRT – 5A**
**Rationalizing Denominators: Single-Term Radical in the Denominator**

Rationalize the denominator. Assume all variables represent positive real numbers.

| | |
|---|---|
| **1.** $\frac{5}{\sqrt{3}}$ | **2.** $\frac{2}{\sqrt{8}}$ |
| **3.** $\frac{3a}{\sqrt{5a}}$ | **4.** $\frac{6x^2}{\sqrt{9x}}$ |
| **5.** $\frac{\sqrt{15}}{2\sqrt{21}}$ | **6.** $\frac{n\sqrt{6}}{12\sqrt{3n}}$ |
| **7.** $\frac{9\sqrt{10}}{x\sqrt{27}}$ | **8.** $\frac{4ab\sqrt{3}}{21\sqrt{6ab^3}}$ |

**Elementary Algebra**
**Skill-Builder # SQRT – 5A**
**Rationalizing Denominators: Single-Term Radical in the Denominator**

**Answers**

| | |
|---|---|
| **1.** $\frac{5\sqrt{3}}{3}$ | **2.** $\frac{\sqrt{2}}{2}$ |
| **3.** $\frac{3a\sqrt{5a}}{5a}$ | **4.** $2x\sqrt{x}$ |
| **5.** $\frac{\sqrt{35}}{14}$ | **6.** $\frac{\sqrt{2n}}{12}$ |
| **7.** $\frac{\sqrt{30}}{x}$ | **8.** $\frac{2b\sqrt{2ab}}{21}$ |

Prepared by: Teresa V. Sutcliffe, Fall 2012