Elementary Algebra
Skill-Builder # SQRT – 3B
Multiplying Square Root Radicals: Multi-Term Radicals

To multiply square root radicals involving several terms simply apply the distributive property for multiplication over addition.

Examples Perform the multiplication.

1. $\sqrt{5}\left(\sqrt{7} + 2\sqrt{2} - 3\sqrt{3}\right)$

Solution: Distribute $\sqrt{5}$ to each of the three terms inside the second factor to get

$$\sqrt{5}\left(\sqrt{7} + 2\sqrt{2} - 3\sqrt{3}\right) = \sqrt{5}\sqrt{7} + \sqrt{5}\cdot 2\sqrt{2} - \sqrt{5}\cdot 3\sqrt{3} = \sqrt{35} + 2\sqrt{10} - 3\sqrt{15}.$$

2. $\left(4\sqrt{2} + 5\right)\left(\sqrt{7} - \sqrt{3}\right)$

Solution: Now it looks like we are multiplying a radical expression with two terms to another with two terms. We can think of this as "binomial" multiplication and we can apply FOIL as follows:

$$\left(4\sqrt{2} + 5\right)\left(\sqrt{7} - \sqrt{3}\right) = 4\sqrt{2}\cdot\sqrt{7} - 4\sqrt{2}\cdot\sqrt{3} + 5\sqrt{7} - 5\sqrt{3} = 4\sqrt{14} - 4\sqrt{6} + 5\sqrt{7} - 5\sqrt{3}.$$

3. $\left(\sqrt{8} + \sqrt{5}\right)^2$

Solution: We can perform the squaring using the special product formula $\left(a + b\right)^2 = a^2 + 2ab + b^2$

or we can write $\left(\sqrt{8} + \sqrt{5}\right)^2$ as $\left(\sqrt{8} + \sqrt{5}\right)\left(\sqrt{8} + \sqrt{5}\right)$ and do FOIL. Let's do the first one:

$$\left(\sqrt{8} + \sqrt{5}\right)^2 = \left(\sqrt{8}\right)^2 + 2\sqrt{8}\cdot\sqrt{5} + \left(\sqrt{5}\right)^2 = 8 + 2\sqrt{40} + 5 = 13 + 2\sqrt{4\cdot 10} = 13 + 4\sqrt{10}.$$

We can of course have variables and again we will assume these represent positive real numbers.

4. $\left(\sqrt{x} - 2\sqrt{y}\right)\left(\sqrt{x} + 2\sqrt{y}\right)$

Solution: We note that the product looks like the special form $\left(a + b\right)\left(a - b\right)$ which gives

$a^2 - b^2$ so using this, we get

$$\left(\sqrt{x} - 2\sqrt{y}\right)\left(\sqrt{x} + 2\sqrt{y}\right) = \left(\sqrt{x}\right)^2 - \left(2\sqrt{y}\right)^2 = x - 4y.$$

Note that we could also just have applied FOIL and gotten the same answer.

We are of course not limited to binomial multiplication.

5. $\left(3\sqrt{2} + 2\sqrt{6}\right)\left(\sqrt{3} - 3\sqrt{5} + 2\sqrt{2}\right)$

Solution: Apply the distributive property of multiplication over addition to get
$$\left(3\sqrt{2} + 2\sqrt{6}\right)\left(\sqrt{3} - 3\sqrt{5} + 2\sqrt{2}\right) = 3\sqrt{2}\cdot\sqrt{3} - 3\sqrt{2}\cdot 3\sqrt{5} + 3\sqrt{2}\cdot 2\sqrt{2}$$

$$+ 2\sqrt{6}\cdot\sqrt{3} - 2\sqrt{6}\cdot 3\sqrt{5} + 2\sqrt{6}\cdot 2\sqrt{2}$$

$$= 3\sqrt{6} - 9\sqrt{10} + 6\sqrt{4} + 2\sqrt{18} - 6\sqrt{30} + 4\sqrt{12}$$

$$= 3\sqrt{6} - 9\sqrt{10} + 12 + 6\sqrt{2} - 6\sqrt{30} + 8\sqrt{3}$$

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Find the following. Assume all variables represent positive real numbers.

1. $2\sqrt{7}\left(\sqrt{5} - \sqrt{3} + 5\sqrt{2}\right)$	**2.** $3\sqrt{a}\left(2\sqrt{3a} + a\sqrt{2} - 3\sqrt{a}\right)$
3. $\left(\sqrt{2} + \sqrt{5}\right)\left(\sqrt{10} + \sqrt{15}\right)$	**4.** $\left(3\sqrt{x} - 2\sqrt{y}\right)\left(\sqrt{x} + 4\sqrt{y}\right)$
5. $\left(\sqrt{8} + \sqrt{12}\right)^2$	**6.** $\left(\sqrt{5n} - 5\sqrt{2}\right)^2$
7. $\left(3\sqrt{8} - 2\sqrt{3}\right)\left(3\sqrt{8} + 2\sqrt{3}\right)$	**8.** $\left(4x - 6\sqrt{2y}\right)\left(4x + 6\sqrt{2y}\right)$

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Answers

1. $2\sqrt{35} - 2\sqrt{21} + 10\sqrt{14}$	**2.** $6a\sqrt{3} + 3a\sqrt{2a} - 9a$
3. $2\sqrt{5} + \sqrt{30} + 5\sqrt{2} + 5\sqrt{3}$	**4.** $3x + 10\sqrt{xy} - 8y$
5. $20 + 8\sqrt{6}$	**6.** $5n - 10\sqrt{10n} + 50$
7. 60	**8.** $16x^2 - 72y$

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