

**Elementary Algebra**  
**Skill-Builder # PF – 4B**

**Factoring Quadratic Trinomials with Leading Coefficient Different from 1:**  
**Bottoms-up Method**

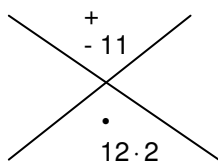
We now show how to factor trinomials of the form  $ax^2 + bx + c$ , where the leading coefficient  $a \neq 1$ , using the bottoms-up method. The method starts just like in the grouping method: one thinks of factors of the product  $ac$  that add up to the middle coefficient  $b$ . One then proceeds by dividing each number by the leading coefficient  $a$  and reducing the resulting fraction. If the fractions in lowest terms are say,  $\frac{p}{q}$  and  $\frac{r}{s}$ , and let's say the variable is  $x$ , then the factored form is  $(qx + p)(sx + r)$ . If there is a second variable, say  $y$ , then the factored form becomes  $(qx + py)(sx + ry)$ .

We will show how to do the examples in the previous skill-builder (#PF – 4A) using this technique.

**Examples** Factor the following.

1.  $12x^2 - 11x + 2$

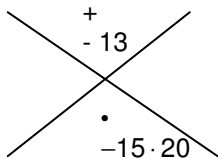
Solution: Start with the 'X' diagram as follows:



We see that  $-8$  and  $-3$  are the factors of  $12 \cdot 2$  or  $24$  which will add up to  $-11$ . We divide these two numbers by  $12$  to get  $-\frac{8}{12}$  and  $-\frac{3}{12}$ . Reduce these fractions to  $-\frac{2}{3}$  and  $-\frac{1}{4}$ . Apply the bottoms-up technique to get the factored form  $(3x - 2)(4x - 1)$ .

2.  $15a^2 - 13ab - 20b^2$

Solution: We can start with the diagram



From SB #PF – 4A we saw that the numbers that will give the product  $-15 \cdot 20$  or  $-300$  and the sum  $-13$  are  $-25$  and  $12$ . We divide these by the leading coefficient  $15$  to get  $-\frac{25}{15}$  and  $\frac{12}{15}$ .

Reduce each to get  $-\frac{5}{3}$  and  $\frac{4}{5}$ . Do bottoms-up to arrive at the factored form  $(3a - 5b)(5a + 4b)$ .

**NOTE:** It is important to factor out any GCF's first BEFORE applying the bottoms-up technique.

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Factor using the bottoms-up method.

1. $2x^2 + 13x + 20$	2. $3y^2 - 13y + 4$
3. $12n^2 - 8n - 15$	4. $9m^2 + 6m - 8$
5. $10x^2 - 37xy + 7y^2$	6. $18a^2 - 15ab - 25b^2$
7. $6w^2 + wy - 40y^2$	8. $12n^2 + 19nm + 4m^2$

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**Answers**

1. $(2x+5)(x+4)$	2. $(y-4)(3y-1)$
3. $(6n+5)(2n-3)$	4. $(3m+4)(3m-2)$
5. $(5x-y)(2x-7y)$	6. $(6a+5b)(3a-5b)$
7. $(2w-5y)(3w+8y)$	8. $(4n+m)(3n+4m)$

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