

**Elementary Algebra**  
**Skill-Builder # PF – 6**  
**Factoring the Sum or Difference of Two Cubes**

We consider polynomials of the form  $a^3 + b^3$  or  $a^3 - b^3$ , or those polynomials that look like the sum or difference of two cubes. Let us first consider the sum  $a^3 + b^3$ . A natural factor one can think of is the binomial  $a + b$ . Then if one thinks that one needs to multiply  $a$  by something to get  $a^3$ , then that something would have to be  $a^2$ ; similarly, one would need a  $b^2$  to get  $b^3$ . So, it would seem that we would need something like

$$(a + b)(a^2 + b^2).$$

However, if we apply FOIL to the above we would get

$$a^3 + ab^2 + a^2b + b^3$$

and we have two extra terms  $ab^2$  and  $a^2b$  that we do not want. To take care of these two terms, we modify the second factor by putting the term  $-ab$ . Verify that

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2).$$

Likewise,

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

**Examples** Factor the following.

**1.**  $x^3 + 1$

Solution:

$$\begin{aligned}x^3 + 1 &= x^3 + 1^3 \\ &= (x + 1)(x^2 - x + 1^2) = (x + 1)(x^2 - x + 1)\end{aligned}$$

**2.**  $n^3 - 8$

Solution:

$$\begin{aligned}n^3 - 8 &= n^3 - 2^3 \\ &= (n - 2)(n^2 + 2n + 2^2) = (n - 2)(n^2 + 2n + 4)\end{aligned}$$

**3.**  $27a^3 + 8b^3$

Solution:

$$\begin{aligned}27a^3 + 8b^3 &= (3a)^3 + (2b)^3 \\ &= (3a + 2b)[(3a)^2 - (3a)(2b) + (2b)^2] \\ &= (3a + 2b)(9a^2 - 6ab + 4b^2)\end{aligned}$$

**Elementary Algebra**  
**Skill-Builder # PF – 6**  
**Factoring the Sum or Difference of Two Cubes**

Factor.

|                   |                                       |
|-------------------|---------------------------------------|
| 1. $a^3 - 1$      | 2. $b^3 + 8$                          |
| 3. $27 - w^3$     | 4. $c^3 + 64$                         |
| 5. $8x^3 - 27$    | 6. $64n^3 - 125$                      |
| 7. $27y^3 - 8x^3$ | 8. $\frac{1}{8}a^3 + \frac{1}{27}b^3$ |

**Elementary Algebra**  
**Skill-Builder # PF – 6**  
**Factoring the Sum or Difference of Two Cubes**

**Answers**

|                             |   |
|-----------------------------|---|
| 1. $(a-1)(a^2+a+1)$         | 2. $(b+2)(b^2-2b+4)$  |
| 3. $(3-w)(9+3w+w^2)$        | 4. $(c+4)(c^2-4c+16)$   |
| 5. $(2x-3)(4x^2+6x+9)$      | 6. $(4n-5)(16n^2+20n+25)$   |
| 7. $(3y-2x)(9y^2+6xy+4x^2)$ | 8. $\left(\frac{1}{2}a+\frac{1}{3}b\right)\left(\frac{1}{4}a^2-\frac{1}{6}ab+\frac{1}{9}b^2\right)$ |

Prepared by: Teresa V. Sutcliffe, Fall 2012