Elementary Algebra
Skill-Builder # PF – 1A
Factoring out the GCF I

To factor out the greatest common factor (GCF), apply the distributive property of multiplication over addition/subtraction in reverse:

$$ab \pm ac = a(b \pm c) \quad \text{or} \quad ab \pm ac = (b \pm c)a$$

Examples Factor the following.

1. $4x + 4y$

Solution: The factor 4 is common to the two terms, thus

$$\boxed{4} \cdot x + \boxed{4} \cdot y = 4(x + y).$$

2. $ab^2y + 2ab^2z - 9ab^2w$

Solution: The factor ab^2 is common to the three terms, thus

$$\boxed{ab^2} \cdot y + 2 \cdot \boxed{ab^2} \cdot z - 9 \cdot \boxed{ab^2} \cdot w = ab^2(y + 2z - 9w).$$

Next, let's consider the case when the GCF is not obvious.

3. $12x^2y + 20x^3y^2$

Solution: We can use the method of prime factorization to find the GCF.

$$\begin{array}{rcl} 12x^2y & = & \boxed{2 \cdot 2} \cdot 3 \cdot \boxed{x \cdot x} \cdot \boxed{y} \\ 20x^3y^2 & = & \boxed{2 \cdot 2} \cdot 5 \cdot \boxed{x \cdot x} \cdot x \cdot \boxed{y} \cdot y \\ \hline \text{GCF} & = & \boxed{2 \cdot 2} \cdot \boxed{x \cdot x} \cdot \boxed{y} = 4x^2y \end{array}$$

Note that for the variable factors, we pick the lowest exponent for the variable. In the example, we chose x^2 over x^3 and y over y^2. Thus,

$$12x^2y + 20x^3y^2 = \boxed{4x^2y} \cdot 3 + \boxed{4x^2y} \cdot 5xy = 4x^2y(3 + 5xy).$$

One may, of course, omit the middle step and work as follows:

$$12x^2y + 20x^3y^2 = 4x^2y(\_\_\_\_ + \_\_\_\_)$$

One can then perform a mental division or mentally figure out the missing factor for each term to arrive at the same answer.

4. $30ab^3c - 48a^3bc^2 + 60a^3b^3c^3$

Solution: Using prime factorization for the numerical coefficients and the observation we made about the variable factors, we get $3abc$ for the GCF.

$$\begin{aligned} & 30ab^3c - 48a^3bc^2 + 60a^3b^3c^3 \\ &= \boxed{3abc} \cdot 10b^2 - \boxed{3abc} \cdot 16a^2c + \boxed{3abc} \cdot 20a^2b^2c^2 \\ &= 3abc(10b^2 - 16a^2c + 20a^2b^2c^2) \end{aligned}$$

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Factor out the greatest common factor (GCF):

1. $5a+5b$	**2.** $2xy+2xz+2xw$
3. $3xy^2-6xy$	**4.** $-5yz^3-10y^2z^3$
5. $8b^3c^2-12b^2c^4$	**6.** $-16x^4y^3+20x^5y^2$
7. $30m^3np^2-12m^2n^4p^3-6m^2np^2$	**8.** $18x^2y^4z^3-9x^2y^2z^2+36x^4y^3z^5$

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Answers

1. $5(a+b)$	**2.** $2x(y+z+w)$
3. $3xy(y-2)$	**4.** $-5yz^3(1+2y)$
5. $4b^2c^2(2b-3c^2)$	**6.** $4x^4y^2(-4y+5x)$ or $-4x^4y^2(4y-5x)$
7. $6m^2np^2(5m-2n^2p-1)$	**8.** $9x^2y^2z^2(2y^2z-1+4x^2yz^3)$

Prepared by: Teresa V. Sutcliffe, Fall 2012