Elementary Algebra
Skill-Builder # PF – 5
Factoring the Difference of Two Squares

We consider polynomials of the form $a^2 - b^2$, or those polynomials that look like the difference of two squares. Let's take a look at, say, $x^2 - 16$. Note that the polynomial is a binomial consisting of terms that are perfect squares. We can also consider the polynomial as a quadratic trinomial whose middle coefficient is 0, i.e. $x^2 - 16 = x^2 + 0x - 16$. If we follow the method we used for factoring such a trinomial, which is to find the factors of -16 that add up to 0, then we see that the numbers we need are -4 and 4. We can generalize this to the following formula for factoring the difference of squares:

$$(a-b)(a+b) = a^2 - b^2.$$

Examples Factor the following.

1. $x^2 - 25$
Solution:

$$x^2 - 25 = x^2 - 5^2 = (x-5)(x+5)$$

2. $9y^2 - 4x^2$
Solution:

$$9y^2 - 4x^2 = (3y)^2 - (2x)^2 = (3y+2x)(3y-2x)$$

3. $\dfrac{a^2}{36} - \dfrac{b^2}{25}$
Solution:

$$\frac{a^2}{36} - \frac{b^2}{25} = \left(\frac{a}{6}\right)^2 - \left(\frac{b}{5}\right)^2 = \left(\frac{a}{6} - \frac{b}{5}\right)\left(\frac{a}{6} + \frac{b}{5}\right)$$

4. $4x^2 + y^2$
Solution:

$4x^2 + y^2$ is of the form $a^2 + b^2$, a sum of squares. We see that no matter how we try, we will not be able to find two binomials that will give us this product. In this case, we say that the polynomial is **prime** or **irreducible** or is **not factorable**.

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Factor.

1. $a^2 - 9$	**2.** $b^2 - 49$
3. $16 - w^2$	**4.** $c^2 + 100$
5. $25x^2 - 81$	**6.** $64n^2 - \frac{1}{9}$
7. $\frac{x^2}{25} - \frac{y^2}{121}$	**8.** $\frac{1}{4}a^2 - \frac{9}{25}b^2$

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Answers

1. $(a-3)(a+3)$	**2.** $(b-7)(b+7)$
3. $(4-w)(4+w)$	**4.** prime
5. $(5x-9)(5x+9)$	**6.** $\left(8n-\frac{1}{3}\right)\left(8n+\frac{1}{3}\right)$
7. $\left(\frac{x}{5}-\frac{y}{11}\right)\left(\frac{x}{5}+\frac{y}{11}\right)$	**8.** $\left(\frac{1}{2}a-\frac{3}{5}b\right)\left(\frac{1}{2}a+\frac{3}{5}b\right)$

Prepared by: Teresa V. Sutcliffe, Fall 2012