Elementary Algebra
Skill-Builder # PF – 7
Factoring Using Combined Techniques

Following is a general strategy one can follow when faced with a factoring problem:

1. Factor out any greatest common factor (GCF).
2. Count the terms.
   - **a.** Two terms – check if the terms are perfect squares or perfect cubes and use one of the following:
     - **i.** $a^2 - b^2 = (a-b)(a+b)$
     - **ii.** $a^2 + b^2$ prime or not factorable
     - **iii.** $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
     - **iv.** $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
   - **b.** Three terms – check if the polynomial is of the form $ax^2 + bx + c$ and do the following:
     - **i.** If $a = 1$, find the factors of *c* that add up to *b*. Let's say these are *m* and *n*, then the factored form is $(x+m)(x+n)$.
     - **ii.** If $a \neq 1$, find the factors of *ac* that add up to *b*. Then use either the grouping method or the bottoms-up method. Of course, one can always do the trial-and-error method.
   - **c.** Four terms – do factoring by grouping

Examples Factor the following.

1. $12x^2 - 75y^2$

Solution: We note that 3 is the GCF of the two terms so we can factor it out:

$$12x^2 - 75y^2 = 3(4x^2 - 25y^2)$$

Now, the two terms inside the quantity are both perfect squares so we use the formula for factoring the difference of squares to get:

$$3(4x^2 - 25y^2) = 3(2x-5y)(2x+5y)$$

2. $n^4 - 5n^2 + 4$

Solution: The terms do not have any GCF so we proceed by counting the terms and we see that there are three. We next check if we can write the trinomial in the form $ax^2 + bx + c$. Now, we observe that $n^4 = (n^2)^2$, so if we do the substitution $x = n^2$, we see that we can rewrite the trinomial as $x^2 - 5x + 4$, a quadratic trinomial with leading coefficient of 1. So, we now find factors of 4 that will give -5 when added and these are -1 and -4. We then have

$$x^2 - 5x + 4 = (x-1)(x-4).$$

But *x* is really n^2, so if we substitute n^2 back, we get

$$(n^2 - 1)(n^2 - 4).$$

Now, we note that each factor is a binomial and each term is a perfect square and that both binomials look like a difference of squares. So, we factor each as a difference of squares to get

$$(n-1)(n+1)(n-2)(n+2)$$

for the complete factored form of $n^4 - 5n^2 + 4$.

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Factor.

1. $18a^3 - 8ab^2$	**2.** $16a^3 + 54$
3. $4t^3 + 24t^2 + 36t$	**4.** $x^2y^2 - 9y^2 - 4x^2 + 36$
5. $x^4 - 13x^2 + 36$	**6.** $a^5 - a^3 + 8a^2 - 8$
7. $10y^4 - 21y^3 - 10y^2$	**8.** $9x^4 - 37x^2 + 4$

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Answers

1. $2a(3a-2b)(3a+2b)$	**2.** $2(2a+3)\left(4a^2-6a+9\right)$
3. $4t(t+3)^2$	**4.** $(x-3)(x+3)(y-2)(y+2)$
5. $(x-2)(x+2)(x-3)(x+3)$	**6.** $(a-1)(a+1)(a+2)\left(a^2-2a+4\right)$
7. $y^2(2y-5)(5y+2)$	**8.** $(3x-1)(3x+1)(x-2)(x+2)$

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