THEOREMS AND COROLLARIES:

1.3.1 The midpoint of a line segment is unique.
1.4.1 There is one and only one angle bisector for a given angle.
1.6.1 There is exactly one line perpendicular to a given line at any point on the line.
1.6.2 The perpendicular bisector of a line segment is unique
1.7.1 If two lines are perpendicular, then they meet to form right angles.
1.7.2 If two lines meet to form a right angle, then these lines are perpendicular.
1.7.3 If two angles are complementary to the same angle (or to congruent angles), then these angles are congruent.
1.7.4 If two angles are supplementary to the same angle (or to congruent angles), then these angles are congruent.
1.7.5 If two lines intersect, then the vertical angles formed are congruent.
1.7.6 Any two right angles are congruent.
1.7.7 If the exterior sides of two adjacent acute angles form perpendicular rays, then these angles are complementary.
1.7.8 If the exterior sides of two adjacent angles form a straight line, then these angles are supplementary.
1.7.9 If two segments are congruent, then their midpoints separate these segments into four congruent segments.
1.7.10 If two angles are congruent, then their bisectors separate these angles into four congruent angles.
2.1.1 From a point not on the given line, there is exactly one line perpendicular to the given line.
2.1.2 If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.
2.1.3 If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.
2.1.4 If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.
2.1.5 If two parallel lines are cut by a transversal, then the exterior angles on the same side of the transversal are supplementary.
2.3.1 If two lines are cut by a transversal so that the corresponding angles are congruent, then these lines are parallel.
2.3.2 If two lines are cut by a transversal so that the alternate interior angles are congruent, then these lines are parallel.
2.3.3 If two lines are cut by a transversal so that the alternate exterior angles are congruent, then these lines are parallel.
2.3.4 If two lines are cut by a transversal so that the interior angles on the same side of the transversal are supplementary, then these
lines are parallel.
2.3.5 If two lines are cut by a transversal so that the exterior angles on the same side of the transversal are supplementary, then these lines are parallel.
2.3.6 If two lines are both parallel to a third line, then these lines are parallel to each other.
2.3.7 If two coplanar lines are both perpendicular to a third line, then these lines are parallel to each other.
2.4.1 In a triangle, the sum of the measures of the interior angles is 180°.
2.4.2 Each angle of an equiangular triangle measures 60°.
2.4.3 The acute angles of a right triangle are complementary.
2.4.4 If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.
2.4.5 The measure of an exterior angle of a triangle equals the sum of the measures of the two non-adjacent interior angles.
2.5.1 The total number of diagonals D in a polygon of n sides is given by the formula $D = \frac{n(n-3)}{2}$.
2.5.2 The sum S of the measures of the interior angles of a polygon with n sides is given by the formula $S = (n-2)\cdot 180^\circ$.
2.5.3 The measure I of each interior angle of a regular polygon of n sides is $I = \frac{(n-2)\cdot 180^\circ}{n}$.
2.5.4 The sum of the measures of the four interior angles of a quadrilateral is 360°.
2.5.5 The sum of the measures of the exterior angles, one at each vertex, of a polygon is 360°.
2.5.6 The measure E of each exterior angle of a regular polygon of n sides is $E = \frac{360^\circ}{n}$.
3.1.1 (**AAS**) If two angles and a non-included side of one triangle are congruent to angles and a non-included side of a second triangle, then the triangles are congruent.
3.2.1 (**HL**) If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent.
3.3.1 Corresponding altitudes of congruent triangles are congruent.
3.3.2 The bisector of the vertex angle of an isosceles triangle separates the triangle into two congruent triangles.
3.3.3 If two sides of a triangle are congruent, then the angles opposite these sides are also congruent.
3.3.4 If two angles of a triangle are congruent, then the sides opposite these angles are also congruent.
3.3.5 An equilateral triangle is also equiangular.
3.3.6 An equiangular triangle is also equilateral.
3.5.1 The measure of a line segment is greater than the measure of any of its parts.
3.5.2 The measure of an angle is greater than the measure of any of its parts.
3.5.3 The measure of an exterior angle of a triangle is greater than the measure of either non-adjacent interior angle.
3.5.4 If a triangle contains a right or an obtuse angle, then the measure of this angle is greater than the measure of either of the remaining angles.
3.5.5 (**Addition Property of Inequality**) If $a > b$ and $c > d$, then $a + c > b + d$.
3.5.6 If one side of a triangle is longer than a second side, then the measure of the angle opposite the first side is greater than the measure of the angle opposite the second side.
3.5.7 If the measure of one angle of a triangle is greater than the measure of a second angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.
3.5.8 The perpendicular segment from a point to a line is the shortest segment that can be drawn from the point to the line.
3.5.9 The perpendicular segment from a point to a plane is the shortest segment that can be drawn from the point to the plane.
3.5.10 (**Triangle Inequality**) The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
3.5.10 (**Alternative**) The length of one side of a triangle must be between the sum and the difference of the lengths of the other two sides.
4.1.1 A diagonal of a parallelogram separates it into two congruent triangles.
4.1.2 Opposite angles of a parallelogram are congruent.
4.1.3 Opposite sides of a parallelogram are congruent.
4.1.4 Diagonals of a parallelogram bisect each other.
4.1.5 Consecutive angles of a parallelogram are supplementary.
4.1.6 If two sides of one triangle are congruent to two sides of a second triangle and the included angle of the first triangle is greater than the included angle of the second triangle, then the length of the side opposite the included angle of the first triangle is greater than the length of the side opposite the included angle of the second.
4.1.7 In a parallelogram with unequal pairs of consecutive angles, the longer diagonal lies opposite the obtuse angle.
4.2.1 If two sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.
4.2.2 If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram.
4.2.3 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
4.2.4 In a kite, one pair of opposite angles are congruent.
4.2.5 The segment that joins the midpoints of two sides of a triangle is parallel to the third side and has a length equal to one-half the length of the third side.
4.3.1 All angles of a rectangle are right angles.
4.3.2 The diagonals of a rectangle are congruent.
4.3.3 All sides of a square are congruent.
4.3.4 All sides of a rhombus are congruent.
4.3.5 The diagonals of a rhombus are perpendicular.
4.4.1 The base angles of an isosceles trapezoid are congruent.
4.4.2 The diagonals of an isosceles trapezoid are congruent.
4.4.3 The length of the median of a trapezoid equals one-half the sum of the lengths of the two bases.
4.4.4 The median of a trapezoid is parallel to each base.
4.4.5 If two (of three) consecutive angles of a quadrilateral are supplementary, the quadrilateral is a trapezoid.
4.4.6 If two base angles of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.
4.4.7 If the diagonals of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.
4.4.8 If three (or more) parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on any transversal.
5.2.1 (**AA**) If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.
5.2.2 The lengths of the corresponding altitudes of similar triangles have the same ratio as the lengths of any pair of corresponding sides.
5.2.3 (**SAS~**) If an angle of one triangle is congruent to an angle of a second triangle and the pairs of sides including these angles are proportional, then the triangles are similar.
5.2.4 (**SSS~**) If the three sides of one triangle are proportional (in length) to the three sides of a second triangle, then the triangles are similar.
5.3.1 The altitude drawn to the hypotenuse of a right triangle separates the right triangle into two right triangles that are similar to each other and to the original right triangle.
5.3.2 The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.
5.3.3 The length of each leg of a right triangle is the geometric mean of the hypotenuse and the length of the segment of the hypotenuse adjacent to that leg.
5.3.4 (**Pythagorean Theorem**) The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs.
5.3.5 (**Converse of Pythagorean Theorem**) If *a*, *b*, and *c* are the lengths of the three sides of a triangle, with c the length of the longest side, and if $c^2 = a^2 + b^2$, then the triangle is a right triangle with the right angle opposite the side of length c.
5.3.6 (**HL**) If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent.

5.3.7 Let *a*, *b*, and *c* represent the lengths of the three sides of a triangle with *c* the length of the longest side.

1. If $c^2 > a^2 + b^2$, then the triangle is obtuse and the obtuse angle lies opposite the side of length *c*.

2. If $c^2 < a^2 + b^2$, then the triangle is acute.

5.4.1 (**45-45-90 Theorem**) In a triangle, whose angles measure 45°, 45°, and 90°, the hypotenuse has a length equal to the product of $\sqrt{2}$ and the length of either leg.

5.4.2 (**30-60-90 Theorem**) In a triangle whose angles measure 30°, 60°, and 90°, the hypotenuse has a length equal to twice the length of the shorter leg, and the length of the longer leg is the product of $\sqrt{3}$ and the length of the shorter leg.

5.4.3 If the length of the hypotenuse of a right triangle equals the product of $\sqrt{2}$ and the length of either leg, then the angles of the triangle measure 45°, 45°, and 90°.

5.4.4 If the length of the hypotenuse of a right triangle is twice the length of one leg of the triangle, then the angle of the triangle opposite that leg measures 30°.

5.5.1 If a line is parallel to one side of a triangle and intersects the other two sides, then it divides these sides proportionally.

5.5.2 When three (or more) parallel lines are cut by a pair of transversals, the transversals are divided proportionally by the parallel lines.

5.5.3 If a ray bisects one angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the two sides that form that angle.

6.1.1 A radius that is perpendicular to a chord bisects the chord.

6.1.2 The measure of an inscribed angle of a circle is one-half the measure of its intercepted arc.

6.1.3 In a circle or in congruent circles, congruent minor arcs have congruent central angles.

6.1.4 In a circle or in congruent circles, congruent central angles have congruent arcs.

6.1.5 In a circle or in congruent circles, congruent chords have congruent minor (major) arcs.

6.1.6 In a circle or in congruent circles, congruent arcs have congruent chords.

6.1.7 Chords that are at the same distance from the center of a circle are congruent.

6.1.8 Congruent chords are located at the same distance from the center of a circle.

6.1.9 An angle inscribed in a semicircle is a right angle.

6.1.10 If two inscribed angles intercept the same arc, then these angles are congruent.

6.2.1 If a quadrilateral is inscribed in a circle, the opposite angles are supplementary.

6.2.2 The measure of an angle formed by two chords intersecting within a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

6.2.3 The radius (or any other line through the center of a circle) drawn to a tangent at the point of tangency is perpendicular to the tangent at that point.

6.2.4 The measure of an angle formed by a tangent and a chord drawn to the point of tangency is one-half the measure of the intercepted arc.

6.2.5 The measure of an angle formed when two secants intersect at a point outside the circle is one-half the difference of the measures of the two intercepted arcs.

6.2.6 If an angle is formed by a secant and a tangent that intersect in the exterior of the circle, then the measure of the angle is one-half the difference of the measures of the intercepted arcs.

6.2.7 If an angle is formed by two intersecting tangents, then the measure of the angle is one-half the difference of the measures of the intercepted arcs.

6.2.8 If two parallel lines intersect a circle, the intercepted arcs between these lines are congruent.

6.3.1 If a line is drawn through the center of a circle perpendicular to a chord, then it bisects the chord and its arc.

6.3.2 If a line through the center of a circle bisects a chord other than the diameter, then it is perpendicular to the chord.

6.3.3 The perpendicular bisector of a chord contains the center of the circle.

6.3.4 The tangent segments to a circle from an external point are congruent.

6.3.5 If two chords intersect within a circle, then the product of the lengths of the segments (parts) of onechord is equal to the product of the lengths of the segments of the other chord.

6.3.6 If two secant segments are drawn to a circle from an external point, then the products of the lengthsof each secant with its external segment are equal.

6.3.7 If a tangent segment and a secant segment are drawn to a circle from an external point, then the square of the length of the tangent equals the product of the length of the secant with the length of its external segment.

6.4.1 The line that is perpendicular to the radius of a circle at its endpoint on the circle is tangent to the circle.

6.4.2 In a circle (or in congruent circles) containing two unequal central angles, the larger angle corresponds to the larger intercepted arc.

6.4.3 In a circle (or in congruent circles) containing two unequal arcs, the larger arc corresponds to thelarger central angle.

6.4.4 In a circle (or in congruent circles) containing two unequal chords, the shorter chord is at the greater distance from the center of the circle.

6.4.5 In a circle (or in congruent circles) containing two unequal chords, the chord nearer the center of the circle has the greater length.

6.4.6 In a circle (or in congruent circles) containing two unequal chords, the longer chord corresponds to the greater minor arc.

6.4.7 In a circle (or in congruent circles) containing two unequal minor arcs, the greater minor arc corresponds to the longer of the chords related to these arcs.

6.5.1 The locus of points in a plane equidistant from the sides of an angle is the angle bisector.

6.5.2 The locus of points in a plane that are equidistant from the endpoints of a line segment is the perpendicular bisector of that line segment.

6.6.1 The three angle bisectors of the angles of a triangle are concurrent.

6.6.2 The three perpendicular bisectors of the sides of a triangle are concurrent.

6.6.3 The three altitudes of a triangle are concurrent.

6.6.4 The three medians of a triangle are concurrent at a point that is two-thirds the distance from any vertex to the midpoint of the opposite side.

7.1.1 The area A of a square whose sides are each of length s is given by $A = s^2$.

7.1.2 The area A of a parallelogram with a base of length b and with corresponding altitude of length h is given by $A = bh$.

7.1.3 The area A of a triangle whose base has length b and whose corresponding altitude has length h is given by $A = \frac{1}{2}bh$.

7.2.1 (Heron's Formula) If the three sides of a triangle have lengths a, b, and c, then the area A of the triangle is given by $A = \sqrt{s(s-a)(s-b)(s-c)}$ where the semiperimeter of the triangle is $s = \frac{1}{2}(a+b+c)$.

7.2.2 The area A of a trapezoid whose bases has lengths b_1 and b_2 and whose altitude has length h is given by $A = \frac{1}{2}h(b_1 + b_2)$.

7.2.3 The area A of any quadrilateral with perpendicular diagonals of lengths d_1 and d_2 is given by $A = \frac{1}{2}d_1 d_2$.

7.2.4 The area A of a rhombus whose diagonals have lengths d_1 and d_2 is given by $A = \frac{1}{2}d_1 d_2$.

7.2.5 The area A of a kite whose diagonals have lengths d_1 and d_2 is given by $A = \frac{1}{2}d_1 d_2$.

7.2.6 The ratio of the areas of two similar triangles equals the square of the ratio of the lengths of any two corresponding sides; that is $\frac{A_1}{A_2} = \left(\frac{a_1}{a_2}\right)^2$.

7.3.1 A circle can be circumscribed about (or inscribed in) any regular polygon.

7.3.2 The measure of the central angle of a regular polygon of n sides is given by $c = \frac{360}{n}$.

7.3.3 Any radius of a regular polygon bisects the angle at the vertex to which it is drawn.

7.3.4 Any apothem to a side of a regular polygon bisects the side of the polygon to which it is drawn.

7.3.5 The area A of a regular polygon whose apothem has length a and whose perimeter is P is given by $A = \frac{1}{2}aP$.

7.4.1 The circumference of a circle is given by the formula $C = \pi d$ or $C = 2\pi r$.

7.4.2 In a circle whose circumference is C, the length ℓ of an arc whose degree measure is m is given by $\ell = \frac{m}{360} \cdot C$

7.4.3 The area A of a circle whose radius is of length r is given by $A = \pi r^2$.

7.5.1 In a circle of radius r, the area A of a sector whose arc has degree measure m is given by $A = \frac{m}{360}\pi r^2$.

7.5.2 The area of a semicircular region of radius r is $A = \frac{1}{2}\pi r^2$.

7.5.3 Where P represents the perimeter of a triangle and r represents the length of the radius of its inscribed circle, the area of the triangle is given by $A = \frac{1}{2}rP$.