Math 267 Fifth Exam **Supplementary Review Problems**

1. Sketch the vector field $\vec{F}(x,y)=\sqrt{x}\,\vec{i}$ by drawing some typical non-intersecting vectors. The vectors need not be drawn to the same scale as the coordinate axes, but they should be in the correct proportions relative to each other.

2. Find $div\,\vec{F}$ and $curl\,\vec{F}$ of $\vec{F}(x,y)=e^x\cos y\,\vec{i}+e^x\sin y\,\vec{j}$.

3. Sketch the gradient field of $\phi(x,y)=2x-y$.

4. Evaluate the line integral $\int_C \frac{ze^{(z^2+2)}}{x^2+y^2}ds$, where C is the helix $x=\cos t,\ y=\sin t,\ z=t,\ 0\le t\le 2\pi$.

5. Evaluate $\int_C (e^x-3y)dx+(e^y+6x)dy$ where C is the path from $(0,0)$ to $(1,0)$ to $(0,2)$ to $(0,0)$.

6. Evaluate $\int_C \vec{F}\bullet d\vec{r}$ where $\vec{F}(x,y,z)=z\vec{i}+x\vec{j}+y\vec{k}$ and C is the helix
$\vec{r}(t)=\sin t\,\vec{i}+3\sin t\,\vec{j}+\sin^2 t\,\vec{k}$ for $0\le t\le \pi/2$.

7. Find the work done by the force $\vec{F}(x,y,z)=(x^2-y)\vec{i}+(y^2-z)\vec{j}+(z^2-x)\vec{k}$ acting on a particle that moves along the curve given by $y=x^2,\ z=x^3$ from $(0,0,0)$ to $(1,1,1)$.

8. Find the mass of a thin wire shaped in the form of the circular arc $y=\sqrt{4-x^2}$ $(0\le x\le 2)$ if the density function is $f(x,y)=kxy^{3/2}\ (k>0)$.

9. Determine whether $F(x,y)=(2x+y^3)\vec{i}+(3xy^2-e^{-2y})\vec{j}$ is conservative. If it is, find a potential function for it.

10. Show that $\int_{(0,0)}^{(1,\pi/2)}(\sin y+y\sin x)dx+(x\cos y-\cos x)dy$ is independent of path and evaluate.

11. Find the work done by the conservative force
$\vec{F}(x,y)=(y\sec^2 x+\sec x\tan x)\vec{i}+(\tan x+2y)\vec{j}$ as it acts on particle moving from
$P(0,0)$ to $Q(\pi/4,1)$.

12. Find $\int \vec{F}\bullet d\vec{r}$ where $\vec{F}(x,y)=(y+2xe^y)\vec{i}+(x+x^2e^y)\vec{j}$ and C is the curve
$\vec{r}(t)=\sqrt{t}\,\vec{i}+\ln t\,\vec{j}$ for $1\le t\le 4$.

13. Use Green's Theorem to evaluate $\int_C (x^2-\cosh y)dx+(y+\sin x)dy$ where C is the boundary of the region enclosed by $0\le x\le \pi$ and $0\le y\le 1$, traversed in a counterclockwise direction.

14. Use Green's Theorem to evaluate $\int_C 2\tan^{-1}\frac{y}{x}dx+\ln(x^2+y^2)dy$ where C is the boundary of the circle $(x-2)^2+y^2=1$ traversed in a counterclockwise manner.

15. Use a line integral to find the area of the region enclosed by $y = \sin x$, $y = \cos x$, and $x = 0$.

16. A particle, starting at $(1,0)$, traverses the upper semicircle $x^2 + y^2 = 1$ and returns to its starting point along the *x*-axis. Use Green's Theorem to find the work done on the particle by a force $\vec{F}(x,y) = xy^2\vec{i} + \left(\frac{1}{3}x^3 + x^2y\right)\vec{j}$.

17. Evaluate the surface integral $\iint_S y\,dS$, where *S* is the first octant portion of the plane $z = x + y$ inside the elliptic cylinder $4x^2 + 9y^2 = 36$.

18. Evaluate $\iint_S \vec{F}\bullet\vec{n}\,dS$, where $\vec{F}(x,y,z) = -x\vec{i} - 2x\vec{j} + (z-1)\vec{k}$ and *S* is the surface enclosed by that portion of the paraboloid $z = 4 - y^2$ which lies in the first octant and is bounded by the coordinate planes and the plane $y = x$. The surface is oriented by upward unit normals.

19. Use the divergence theorem to evaluate $\iint_S \vec{F}\bullet\vec{n}\,dS$, where $\vec{F}(x,y,z) = x^3\vec{i} + x^2y\vec{j} - x^2z\vec{k}$, $\vec{n}$ is the outer unit normal vector to *S*, and *S* is the surface enclosed by the hemisphere $z = \sqrt{4 - x^2 - y^2}$ and the *xy*-plane.

20. Use Stokes' Theorem to evaluate $\int_C xz\,dx + y^2dy + x^2dz$, where *C* is the intersection of the plane $x + y + z = 5$ and the cylinder $x^2 + \frac{y^2}{4} = 1$.

21. Use Stokes' Theorem to evaluate $\iint_S \left(curl\,\vec{F}\right)\bullet\vec{n}\,dS$, where $\vec{F}(x,y,z) = (z-y)\vec{i} + (z^2+x)\vec{j} + (x^2-y^2)\vec{k}$, and *S* is that portion of the sphere $x^2 + y^2 + z^2 = 4$ for which $z \ge 0$.

Answers

2. $div\,\vec{F} = 2e^x\cos y + 1$; $curl\,\vec{F} = 2e^x\sin y\,\vec{k}$

4. $\frac{\sqrt{2}}{2}\left[e^{4\pi^2+2} - e^2\right]$

5. 9

6. 23/6

7. -29/60

8. $16\sqrt{2}/5k$

9. conservative; $f(x,y) = x^2 + xy^3 + \frac{1}{2}e^{-2y} + C$

10. $1 - \frac{\pi}{2}\cos 1$

11. $1 + \sqrt{2}$

12. 15 + 4ln2

13. $\pi(\cosh 1 - 1)$

14. 0

15. $\sqrt{2} - 1$

16. $\pi/8$

17. $4\sqrt{3}$

18. -6

19. $64\pi/5$

20. $\int_{-2}^{2}\int_{-\sqrt{1-\frac{y^2}{4}}}^{\sqrt{1-\frac{y^2}{4}}} -x\,dx\,dy$ **=…**

21. 8π