Math 267 Exam 5 Notes

vector field in 3D $\vec{F}(x,y,z)=M(x,y,z)\vec{i}+N(x,y,z)\vec{j}+P(x,y,z)\vec{k}$,

Let $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ and $\vec{u}=\vec{r}/\|\vec{r}\|$

$\vec{F}$ is an **inverse square field** if $\vec{F}(x,y,z)=\dfrac{c}{\|\vec{r}\|^2}\vec{u}=\dfrac{c}{\|\vec{r}\|^3}\vec{r}$, where *c* is a scalar.

A vector field $\vec{F}$ is **conservative** if $\vec{F}(x,y,z)=\nabla f(x,y,z)$ for some scalar function *f*.

If $\vec{F}$ is conservative, then the function *f* is called a **potential function** for $\vec{F}$ and $f(x,y,z)$ is the **potential** at the point (x,y,z) ….

Every inverse square vector field is conservative.

$$\text{curl}\vec{F}=\nabla\times\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ M & N & P\end{vmatrix};\qquad \text{div }\vec{F}=\nabla\bullet\vec{F}=\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}+\frac{\partial P}{\partial z}$$

Line integrals in 2D

If a smooth curve *C* is given by $x=g(t),\ y=h(t);\ a\le t\le b$ and if $f(x,y)$ is continuous on a region *D* containing *C*, then

(i) $\int_C f(x,y)\,ds=\int_a^b f(g(t),h(t))\sqrt{[g'(t)]^2+[h'(t)]^2}\;dt$

(ii) $\int_C f(x,y)\,dx=\int_a^b f(g(t),h(t))g'(t)\;dt$

(iii) $\int_C f(x,y)\,dy=\int_a^b f(g(t),h(t))h'(t)\;dt$

The **work** *W* **done by** $\vec{F}$ **along** *C* is defined as

$$W=\int_C M(x,y,z)dx+N(x,y,z)dy+P(x,y,z)dz\ ,$$

or, in vector notation, $W=\int_C \vec{F}\bullet\vec{T}\,ds=\int_C \vec{F}\bullet d\vec{r}$

If $\vec{F}(x,y)=\vec{M}(x,y)\vec{i}+\vec{N}(x,y)\vec{j}$ is continuous on an open connected region *D*, then

$\int_C \vec{F}\bullet d\vec{r}$ is independent of path $\Leftrightarrow$ $\vec{F}$ is conservative; that is, $\vec{F}(x,y)=\nabla f(x,y)$

for some scalar function *f*, and $\int_C M(x,y)dx+N(x,y)dy=\int_{(x_1,y_1)}^{(x_2,y_2)}\vec{F}\bullet d\vec{r}=\left[f\left(x,y\right)\right]_{(x_1,y_1)}^{(x_2,y_2)}$.

D simply connected: $\int_C M(x,y)dx+N(x,y)dy$ is independent of path in *D* $\Leftrightarrow$ $\dfrac{\partial M}{\partial y}=\dfrac{\partial N}{\partial x}$

Green's theorem $\oint_C Mdx+Ndy=\iint\limits_R\left(\dfrac{\partial N}{\partial x}-\dfrac{\partial M}{dy}\right)dA$

If a region *R* in the *xy*-plane is bounded by a piecewise-smooth simple closed curve *C*, then the area *A* of *R* is $A=\oint_C x\,dy=-\oint_C y\,dx=\frac{1}{2}\oint_C xdy-ydx$

Vector form of Green's Theorem $\oint_C \vec{F}\bullet\vec{T}\,ds = \iint_R (\nabla\times\vec{F})\bullet\vec{k}\,dA$

Surface Integral $\iint_S g(x,y,z)dS = \iint_{R_{xy}} g(x,y,f(x,y))\sqrt{\left[f_x(x,y)\right]^2+\left[f_y(x,y)\right]^2+1}\,dA$

$$\iint_S g(x,y,z)dS = \iint_{R_{xz}} g(x,h(x,z),z)\sqrt{\left[h_x(x,z)\right]^2+\left[h_z(x,z)\right]^2+1}\,dA$$

$$\iint_S g(x,y,z)dS = \iint_{R_{yz}} g(k(y,z),y,z)\sqrt{\left[k_y(y,z)\right]^2+\left[k_z(y,z)\right]^2+1}\,dA$$

Flux integral of $\vec{F}$ over S $\iint_S \vec{F}\bullet\vec{n}\,dS$, where $\vec{n}$ to S at the point (x,y,z)

$$\vec{n} = \frac{\nabla g(x,y,z)}{\|g(x,y,z)\|} = \frac{-f_x(x,y)\vec{i} - f_y(x,y)\vec{j} + \vec{k}}{\sqrt{\left[f_x(x,y)\right]^2 + \left[f_y(x,y)\right]^2 + 1}}$$

Divergence Theorem $\iint_S \vec{F}\bullet\vec{n}\,dS = \iiint_Q \nabla\bullet\vec{F}\,dV$

Stokes' Theorem $\oint_C \vec{F}\bullet\vec{T}\,dS = \iint_S (\text{curl }\vec{F})\bullet\vec{n}\,dS = \oint_C \vec{F}\bullet d\vec{r}$

curl $F = 0$ in D $\Leftrightarrow$ if $\oint_C \vec{F}\bullet d\vec{r} = 0$ for every simple closed curve C in D.

The following conditions are equivalent:

(i) $\vec{F}$ is conservative; i.e., $\vec{F} = \nabla f$ for some scalar function *f*.

(ii) $\oint_C \vec{F}\bullet d\vec{r}$ is independent of path in *D.*

(iii) $\oint_C \vec{F}\bullet d\vec{r} = 0$ for every simple closed curve C in D.

(iv) $\vec{F}$ is irrotational, i.e., curl $\vec{F} = \vec{0}$.