Math 267 Fourth Exam **Supplementary Review Problems**

Evaluate the iterated integrals.

1. $\int_0^2 \int_{-y}^{2y} xe^{y^3}\,dx\,dy$

2. $\int_0^1 \int_0^z \int_0^{\sqrt{yz}} x\,dx\,dy\,dz$

Express the iterated integral as an equivalent integral with the order of integration reversed.

3. $\int_0^2 \int_0^{x/2} e^x e^y\,dy\,dx$

4. $\int_0^{\pi} \int_y^{\pi} \frac{\sin x}{x}\,dx\,dy$

Use a double integral to find the area of the region bounded by the given curves.

5. $y = 2x^3$, $2x + y = 4$, $y = 0$

6. $x = y^2$, $x = 4y - y^2$

Sketch the region R whose area is given by the iterated integral.

7. $\int_0^{\pi/2} \int_{\tan(x/2)}^{\sin x} dy\,dx$

8. $\int_{\pi/6}^{\pi/2} \int_a^{a(1+\cos\theta)} r\,dr\,d\theta \ \ (a>0)$

Evaluate the double integral over R using either rectangular or polar coordinates.

9. $\iint_R x^2 \sin y^2\,dA$; R is the region bounded by $y = x^3$, $y = -x^3$, and $y = 8$

10. $\iint_R (4 - x^2 - y^2)\,dA$; R is the sector in the first quadrant bounded by the circle $x^2 + y^2 = 4$ and the coordinate axes.

Use a double integral in rectangular or polar coordinates to find the volume of the solid.

11. The solid in the first octant bounded by the coordinate planes and the plane $3x + 2y + z = 6$.

12. The solid enclosed by the cylinders $y = 3x + 4$ and $y = x^2$ such that $0 \le z \le \sqrt{y}$.

13. Convert to polar coordinates and evaluate: $\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} 4xy\,dy\,dx$.

14. Convert to rectangular coordinates and evaluate: $\int_0^{\pi/2} \int_0^{2a\sin\theta} r \sin 2\theta\,dr\,d\theta \ \ (a>0)$.

Find the area of the region using a double integral in polar coordinates.

15. The region outside the circle $r = \sqrt{2}a$ and inside the lemniscate $r^2 = 4a^2 \cos 2\theta$.

16. The region enclosed by the rose $r = \cos 3\theta$.

Find the area of the surface described.

17. The part of the paraboloid $z = 3x^2 + 3y^2 - 3$ below the xy-plane.

18. The part of the cone $z^2 = x^2 + y^2$ between the planes $z = 1$ and $z = 4$.

Evaluate the triple integral over the given solid G.

19. $\iiint_G x^2 yz\,dV$, G: $0 \le x \le 2$, $-x \le y \le x^2$, and $0 \le z \le x + y$

20. $\iiint\limits_G \sqrt{x^2+y^2}\, dV;\ G: x^2+y^2 \le 16,\ 0 \le z \le 4-y$

Express the volume of G as a triple integral in (a) rectangular coordinates and (b) cylindrical coordinates.

21. $G = \{(x,y,z): x^2+y^2 \le z \le 4x\}$

Find an equivalent integral of the form $\iiint\limits_G (\ )\, dx\, dz\, dy$.

22. $\int_0^1 \int_0^{(1-x)/2} \int_0^{1-x-2y} z\, dz\, dy\, dx$

Change to cylindrical coordinates and then evaluate:

23. $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{(x^2+y^2)^2}^{16} x^2\, dz\, dy\, dx$

Change to spherical coordinates and then evaluate:

24. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \dfrac{1}{1+x^2+y^2+z^2} dz\, dy\, dx$

Find the volume of G.

25. G is the solid enclosed by the "inverted apple" $\rho = a(1+\cos\phi)$.

26. G is the solid enclosed between the surfaces $x = y^2+z^2$ and $x = 1-y^2$.

Find the centroid of the plane region R.

27. The region R is bounded by $y^2 = 4x$ and $y^2 = 8(x-2)$

28. The region R is enclosed by the cardioid $r = a(1+\sin\theta)$

Find the center of gravity of the lamina with density δ .

29. The triangular lamina with vertices $(a,0)$, $(-a,0)$, and $(0,b)$, where $a > 0$ and $b > 0$; and δ(x,y) is proportional to the distance from (x,y) to the y-axis

30. The lamina enclosed by the circle $r = 3\cos\theta$, but outside the cardioid $r = 1+\cos\theta$, and with $\delta(r,\theta)$ proportional to the distance from (r,θ) to the x-axis

Find the mass of the solid G if its density is δ .

31. The solid G is the part of the first octant under the plane $x/a + y/b + z/c = 1$, where a,b,c are positive, and $\delta(x,y,z) = kz$.

32. The spherical solid G is bounded by $\rho = a$ and $\delta(x,y,z)$ is twice the distance from (x,y,z) to the origin.

Find the centroid of G.

33. The solid G is bounded by $y = x^2$, $y = 4$, $z = 0$, and $y + z = 4$.

34. The solid G is part of the sphere $\rho \le a$ lying within the cone $\phi \le \phi_0$, where $\phi_0 \le \pi/2$.

35. The solid G is bounded by the cone with vertex $(0,0,h)$ and base $x^2 + y^2 \le R^2$ in the *xy*-plane.

Answers

1. $\left(e^{8}-1\right)/2$

2. 1/16

3. $\int_{0}^{1}\int_{2y}^{2} e^{x}e^{y}\,dx\,dy$

4. $\int_{0}^{\pi}\int_{0}^{x} \frac{\sin x}{x}\,dy\,dx$

5. $3/2$

6. $8/3$

9. $(1-\cos 64)/3$

10. 2π

11. 6

12. 1453/30

13. 4

14. a^{2}

15. $\frac{2}{3}\left(3\sqrt{3}-\pi\right)a^{2}$

16. $\pi/4$

17. $\left(37\sqrt{37}-1\right)\pi/54$

18. $15\sqrt{2}\,\pi$

19. 245552/3465

20. $512\pi/3$

21. **(a)** $\int_{0}^{4}\int_{-\sqrt{4x-x^{2}}}^{\sqrt{4x-x^{2}}}\int_{x^{2}+y^{2}}^{4x} dz\,dy\,dx$ **;** **(b)** $\int_{-\pi/2}^{\pi/2}\int_{0}^{4\cos\theta}\int_{r^{2}}^{4r\cos\theta} r\,dz\,dr\,d\theta$

22. $\int_{0}^{1/2}\int_{0}^{1-2y}\int_{0}^{1-2y-z} z\,dx\,dz\,dy$

23. 32π

24. $\pi(4-\pi)/8$

25. $8\pi a^{3}/3$

26. $\sqrt{2}\,\pi/4$

27. $(8/5,0)$

28. $(0,\,5a/6)$

29. $(0,b/4)$

30. $(4513/2300,\,0)$

31. $\frac{1}{24}kabc^{2}$

32. $2\pi a^{4}$

33. $(0,12/7,8/7)$

34. $\left(0,0,\frac{3}{8}a(1+\cos\phi_{0}\right)$

35. $(0,0,h/4)$