Math 267 Third Exam **Supplementary Review Problems**

1. Given $f(x) = \ln(x - y)$, determine the domain of *f* and draw a sketch showing as a region in $\mathbb{R}^2$ the set of points in the domain.

2. Determine all points where $f(x,y) = \tan(x + y)$ is continuous.

3. Given $f(x,y) = \sqrt{x^2 + y^2 + 4z^2 - 4}$, determine the domain of *f* and describe the region in $\mathbb{R}^3$ that is the set of points in the domain.

4. Determine whether the limit exists and if it does, evaluate it.

a. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\left(x^2 - y^2\right)^2}{x^2 + y^2}$ **b.** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x}{x^2 + y^2}$

5. Given $f(r,\theta,z) = \dfrac{r^2 \tan\theta}{z}$, find $f_r(r,\theta,z)$, $f_\theta(r,\theta,z)$, and $f_z(r,\theta,z)$.

6. Given $f(x,y) = \cos x + \sin xy - \sin y$, find $f_{xx}(x,y)$, $f_{yy}(x,y)$, and $f_{xy}(x,y)$.

7. Given $f(x,y) = e^{xy} + x^3 y$, find $f_{xxx}(x,y)$ and $f_{yxy}(x,y)$.

8. If $f(x,y) = 5x^2 + 3xy - 4y^2$, find

a. $\Delta f(2,1)$

b. $\Delta f(2,1)$ when $\Delta x = 0.01$ and $\Delta y = -0.03$

c. $df(2, 1, \Delta x, \Delta y)$

d. $df(2, 1, 0.01, -0.03)$

9. Given $u(x,y) = e^{x^2}$, where $x = \ln t$ and $y = \sqrt{t}$, find du/dt by using the chain rule.

10. Given $u = x/y$, where $x = r\cos\theta$ and $y = r\sin\theta$, find $\partial u/\partial r$ by using the chain rule.

11. Assuming that $\sin xyz = x^2 + y^2 + z^2$ defines *z* as a function of *x* and *y*, differentiate implicitly to find $\partial z/\partial y$.

12. The moment of inertia *I* of a cylinder of mass *M* and radius *R* with respect to rotation about its axis is given by $I = MR^2/2$. If the error in *M* is measured to be 10 and the error in *R* is 0.08 when *R* is measured to be 4.0, find the maximum error in *I*.

13. The equivalent resistance R_e ohms of a parallel network consisting of two resistors R_1 and R_2 is given by $R_e = R_1/R_2(R_1 + R_2)$. If R_1 and R_2 are changing at rates of 2 ohms/minute and 5 ohms/minute, respectively, find the rate of change of R_e when R_1 is 1000 ohms and R_2 is 1500 ohms.

14. Find the gradient of the function *f* defined by $f(x,y) = (x + y)/(x - y)$.

15. Find the value of the directional derivative at $(-1,0,2)$ for the function *f* defined by $f(x,y,z) = \ln(x^2 + y^2 + z^2)$ in the direction of $\vec{u} = \left(1/\sqrt{6}\right)\vec{i} + \left(2/\sqrt{6}\right)\vec{j} - \left(1/\sqrt{6}\right)\vec{k}$.

16. Find an equation of the tangent plane and equations of the normal line to the surface $x^2 + y^2 - 4z^2 = 4$ at (2,-2,1).

17. Find all points on the elliptic paraboloid $z = 9x^2 + 4y^2$ at which the normal line is parallel to the line through the points $P(4,-2,5)$ and $Q(-2,-6,4)$.

18. Determine the local extrema, if any, of the function *f* defined by
$f(x,y) = \sin x + \sin y + \sin(x+y)$.

19. Use Lagrange multipliers to find the local extrema of the function *f* defined by
$f(x,y,z) = x^2 + y^2 + z^2$ subject to $x^2 = yz + 1$.

20. A trapezoidal gutter is to be made by bending up the edges of a 10-inch wide piece of aluminum an equal amount on each side. Find the width of the base and the angle θ that will result in maximum carrying capacity.

Answers

1. $\{(x,y) \mid x > y\}$

2. $\{(x,y) \mid x + y \neq (2n+1)\pi/2\}$

3. $\{(x,y,z) \mid x^2 + y^2 + 4z^2 \geq 4\}$; all points on or above the xy-plane which are also on or outside the ellipsoid $x^2 + y^2 + 4z^2 = 4$

4a. 0

4b. DNE

5. $(2r\tan\theta)/z;\ (r^2\sec\theta)/z;\ -(r^2\tan\theta)/z^2$

6. $-\cos x - y^2\sin xy;\ -x^2\sin xy + \sin y;\ \cos xy - xy\sin xy$

7. $y^3e^{xy} + 6y;\ xe^{xy}(xy+2)$

8a. $23\Delta x - 2\Delta y + 3\Delta x\Delta y + 5(\Delta x)^2 - 4(\Delta y)^2$

8b. 0.286

8c. $23\Delta x - 2\Delta y$

8d. 0.29

9. $\dfrac{\sqrt{t}\, e^{\sqrt{t}(\ln^2 t)} \ln t(4 + \ln t)}{2t}$

10. $1/y[\cos\theta - (x/y)\sin\theta]$

11. $\dfrac{2y - xz\cos xyz}{xy\cos xyz - 2z}$

12. 3.224

13. 1.52 ohms/minute

14. $\dfrac{-2y}{(x-y)^2}\,\vec{i} + \dfrac{2x}{(x-y)^2}\,\vec{j}$

15. $-6/5\sqrt{6}$

16. $x - y - 2z = 2;\ \dfrac{x-2}{4} = \dfrac{y+2}{-4} = \dfrac{z-1}{-8}$

17. $(-1/3, -1/2, 2)$

18. relative max $\left(\dfrac{\pi}{3} + 2n\pi, \dfrac{\pi}{3} + 2n\pi, 3 + \sqrt{3}\right)$; relative min $\left(-\dfrac{\pi}{3} + 2n\pi, -\dfrac{\pi}{3} + 2n\pi, -3 - \sqrt{3}\right)$

19. relative min at $(1,0,0), (-1,0,0), f(x,y,z) = 1$; relative max at $(0,-1,1), (0,1,-1), f(x,y,z) = 2$

20. 10/3 in, $\theta = 60^\circ$