Math 267 Second Exam ***More Problems! Answers on page 2***

1. Sketch the graph of $\vec{r}(t) = \sec t\,\vec{i} + \tan t\,\vec{j}$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$, and show the direction of increasing t.

2. Describe the graph of $\vec{r}(t) = \langle 3\cos 2t,\, 2\sin 2t,\, t \rangle$.

3. Describe the graph of $\vec{r}(t) = \langle 3\cos 2t,\, 2\sin 2t,\, 2 \rangle$.

4. Given $\vec{r}(t) = t^2\vec{i} + 4t^{-2}\vec{j}$. Find $r'\left(\frac{\pi}{4}\right)$ and sketch the graph showing both $\vec{r}\left(\frac{\pi}{4}\right)$ and $r'\left(\frac{\pi}{4}\right)$.

5. Evaluate $\int_0^{\pi/2} \langle e^t \sin t,\, te^t \rangle dt$.

6. Find parametric equations for $\vec{r}(t) = \langle 6\sin 2t,\, 6\cos 2t \rangle$ using arc length s as parameter. Use the point on the curve where $t = 0$ as the reference point.

7. Find the unit tangent and unit normal vectors to the curve $\vec{r}(t) = \ln t\,\vec{i} + t\,\vec{j}$ at $t = 2$. Sketch a portion of the curve showing the point of tangency.

8. Find the parametric equations of the line which is perpendicular to $\vec{r}(t) = \langle 5t,\, 6\sin 2t,\, 6\cos 2t \rangle$ at $t = \frac{\pi}{3}$.

9. Find the curvature for the curve defined by $x = 2\cos t$, $y = \cos 2t$ at $t = \pi/4$.

10. Find the radius of curvature for $\vec{r}(t) = \langle 4\sin t,\, 2t - \sin 2t,\, \cos 2t \rangle$ at $t = \frac{\pi}{2}$.

11. Sketch $y^2 = 4x$. Calculate the radius of curvature at $(1,2)$ and sketch the circle of curvature.

12. A particle moves through 3-space in such a way that that its velocity is $\vec{v}(t) = 2\vec{i} - 4t^3\vec{j} + 6\sqrt{t}\,\vec{k}$. Find the coordinates of the particle at $t = 1$ second if the particle was initially at $(1,5,3)$ at $t = 0$ sec.

13. A particle travels along a curve given by $\vec{r}(t) = 3\cos 3t\,\vec{i} - 3\sin 3t\,\vec{j} + 2\vec{k}$. Find the **displacement** and the **distance traveled** by the particle during the time interval $0 \le t \le \frac{\pi}{2}$ seconds.

14. A particle travels along a path given by $\vec{r}(t) = \langle \sqrt{2}\,t,\, e^t,\, e^t \rangle$. Find the scalar and vector, tangential and normal components of the acceleration and the curvature of the path at the point $(0,1,1)$.

15. A shell is fired from ground level at an elevation of 60° and strikes a target 6000 meters away. Calculate the muzzle speed of the shell. Use $g = 9.8\ m/\sec^2$.

Answers

1. right branch of hyperbola $x^2 - y^2 = 1$, arrow going up

2. elliptical helix

3. ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ in the $z = 2$ plane

4.

5. $\left\langle \frac{e^{\pi/2}}{2} + 1 \middle/ 2, \frac{\pi}{2} e^{\pi/2} - e^{\pi/2} + 1 \right\rangle$

6. $x = 6\sin\frac{s}{6}, y = 6\cos\frac{s}{6}$

7. $\vec{T}(2) = \frac{1}{\sqrt{5}}\vec{i} + \frac{2}{\sqrt{5}}\vec{j}; \vec{N}(2) = -\frac{2}{\sqrt{5}}\vec{i} + \frac{1}{\sqrt{5}}\vec{j}$

8. $x = \frac{5\pi}{3}, y = 3\sqrt{3} + \sqrt{3}t, z = -3 - t$

9. $\frac{1}{3\sqrt{3}}$

10. $4/\sqrt{2}$

11. $\rho = 4\sqrt{2}$

12. $(3,4,7)$

13. $\overrightarrow{\Delta r} = -3\vec{i} + 3\vec{j}$; $L = 9\frac{\pi}{2}$ units

14. $a_{\vec{T}} = 1, a_{\vec{T}}\vec{T}(0) = \left\langle \frac{\sqrt{2}}{2}, \frac{1}{2}, \frac{1}{2} \right\rangle; a_{\vec{N}} = \frac{1}{2}, a_{\vec{N}}\vec{N}(0) = \left\langle -\frac{\sqrt{2}}{2}, \frac{1}{2}, \frac{1}{2} \right\rangle; K = \frac{1}{4}$

15. 261 m/sec