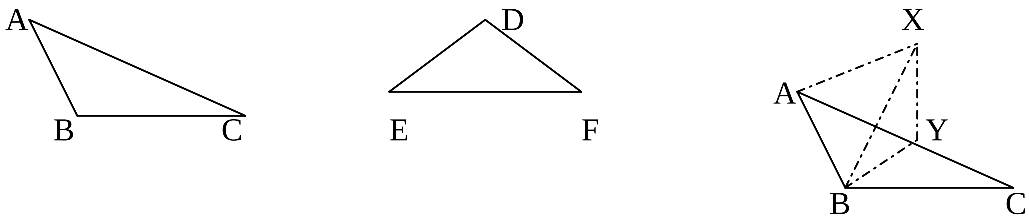
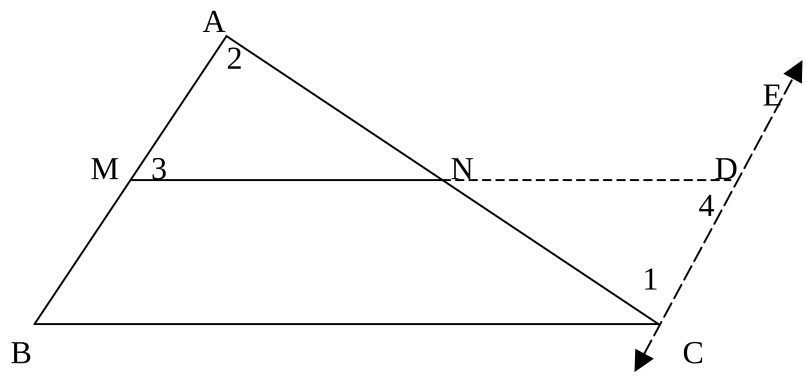
Lemma 4.1.7
If two sides of one triangle are congruent to two sides of a second triangle and the included angle of the first triangle is greater than the included angle of the second, then the length of the side opposite the included angle of the first triangle is greater than the length of the side opposite the included angle of the second.



Given: $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $m\measuredangle B > m\measuredangle E$
Prove: $AC > DF$
Proof:

Statements	Reasons
1. ΔABC, ΔDEF, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $m\measuredangle B > m\measuredangle E$	1. Given
2. Construct $\overline{AX}$ with $\measuredangle ABX \cong \measuredangle DEF$, $\overline{BX} \cong \overline{BC} \cong \overline{EF}$	2. Angle and segment construction
3. $\Delta ABX \cong \Delta DEF$	3. SAS
4. $\overline{AX} \cong \overline{DF}$	4. CPCTC
5. $AX = DF$	5. Congruent sides have equal measures.
6. Construct bisector of $\measuredangle XBC$ cutting $\overline{AC}$ at Y	6. An angle has a unique bisector.
7. $\measuredangle XBY \cong \measuredangle CBY$	7. Definition of angle bisector
8. $\overline{BY} \cong \overline{BY}$	8. Identity
9. $\Delta BXY \cong \Delta BCY$	9. SAS
10. $\overline{XY} \cong \overline{YC}$	10. CPCTC
11. $XY = YC$	11. Congruent sides have equal measures.
12. $AC = AY + YC$	12. Segment-Addition Postulate
13. $AY + YC = AY + XY$ (11)	13. Substitution
14. $AC = AY + XY$ (12), (13)	14. Transitive Property of Equality
15. $AY + XY > AX$	15. Triangle Inequality
16. $AC > DF$ (5), (14), (15)	16. Substitution

Theorem 4.2.5 (Part II)
The length of the segment that joins the midpoints of two sides of a triangle is half the length of the third side of the triangle.



Given: ΔABC with midpoints M and N of $\overline{AB}$ and $\overline{AC}$, respectively
Prove: $MN = \frac{1}{2} BC$

Proof:

Statements	Reasons
1. ΔABC with midpoints M and N of $\overline{AB}$ and $\overline{AC}$, respectively	1. Given
2. Through C, construct $\overleftrightarrow{CE} \parallel \overleftrightarrow{AB}$	2. Parallel Postulate
3. Extend $\overline{MN}$ to meet $\overleftrightarrow{CE}$ at D	3. Exactly one line passes through two points.
4. $\overline{MN} \parallel \overline{BC}$	4. The segment that joins the midpoints of two sides of a triangle is parallel to the third side of the triangle.
5. MBCD is a parallelogram.	5. Definition of parallelogram
6. $MD = BC$	6. Opposite sides of a parallelogram are congruent, hence has equal measures.
7. $\overline{AN} \cong \overline{NC}$	7. Definition of midpoint
8. $\measuredangle 1 \cong \measuredangle 2$, $\measuredangle 3 \cong \measuredangle 4$	8. ITPLACBAT, alternate interior angles are congruent.
9. $\Delta AMN \cong \Delta CDN$	9. AAS
10. $\overline{MN} \cong \overline{ND}$	10. CPCTC
11. $MN = ND$	11. Congruent sides have equal measures.
12. $MD = MN + ND$	12. Segment Addition Postulate
13. $MD = MN + MN = 2MN$ (11), (12)	13. Substitution; Distributive
14. $MN = \frac{1}{2} MD$	14. Division Property
15. $MN = \frac{1}{2} BC$ (6), (14)	15. Substitution