Math 266 Fifth Exam Practice Problems

Sketch and discuss each conic.

1. $x = 4y - y^2$

2. $x^2 - y^2 = 16$

3. $36x^2 + y^2 = 9$

4. $x^2 - 6x - 3y + 15 = 0$

5. $9x^2 + 4y^2 - 18x + 24y + 9 = 0$

6. Find an equation of the parabola with focus at $(0,-5)$ and directrix $y = 5$. Also find the length of the ***latus rectum***, and sketch the graph.

7. Find an equation of the hyperbola with vertices at $(4,2)$ and $(2,2)$ and conjugate axis of length 4, and sketch the graph.

8. Find an equation of the ellipse with center at $(1,4)$, one vertex at $(10,4)$, and minor axis of length 2, and sketch the graph.

Simplify the following by a rotation of axes. Sketch the graph showing the two sets of axes.

9. $21x^2 + 10xy\sqrt{3} + 31y^2 = 144$

10. $17x^2 + 18xy - 7y^2 = 80$

Solve the following.

11. A "whispering gallery" is a room having an elliptical shape and making use of the property that lines drawn from the foci of an ellipse to any point on the ellipse make equal angles with the tangent line at that point. Thus, sound waves generated at one focus of an elliptical room will be collected at the other focus. In such a room 30 feet long and 16 feet wide, where should two people stand so they can hear each other whisper?

12. A parabolic mirror is to be constructed for a large telescope. The mirror must be 4 feet across and must be capable of focusing light exactly 15 feet above its center. Find an equation for the parabola that is the cross section of the mirror, and determine how high the edge of the mirror is above its center.

13. An earth satellite follows an elliptical orbit such that its closest approach to the earth's surface is 1000 miles and its farthest distance is 11,000 miles. If the center of the earth is at one focus of the elliptical orbit, find the lengths of the major and minor axes of the ellipse. (Assume that the earth is a sphere of radius 4000 miles.)

14. Find all horizontal and vertical tangent lines of the graph of the parametric equations

$$x = t^2 - 2t \text{ and } y = 4t - t^2$$

and sketch the graph.

15. Find the length of arc of the curve having the parametric equations

$$x = 2t^2 \text{ and } y = \frac{4}{3}t^3$$

from $t = 0$ to $t = 3$.

16. Find a Cartesian equation of the graph of $r(\cos\theta - 2) = 1$.

17. Find a polar equation of the graph of $4xy + 1 = 0$.

18. Sketch the graph of $r = 2(1 - 2\sin\theta)$.

19. Find all points of intersection of the graphs of $r = 3/2$ and $4r\cos\theta = 3$.

20. Find the area of one loop of the three-leafed rose $r = 4\cos 3\theta$.

21. Find the area of the region inside the graph of $r = 2\sin^2\theta$ bur outside the graph of $r = 1 - \sin\theta$.

22. $r = \dfrac{2}{1+\cos\theta}$ is the equation of a conic having a focus at the pole. Find the eccentricity, identify the conic, write an equation of the directrix that corresponds to the focus at the pole, and draw a sketch of the curve.

23. Find a polar equation of the conic having a focus at the pole and vertices at $\left(1, \dfrac{\pi}{2}\right)$ and $\left(3, \dfrac{\pi}{2}\right)$.