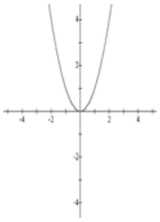
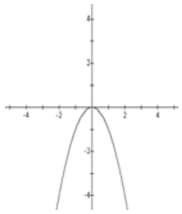
Math 266 Fifth Exam Review Notes

Parabolas



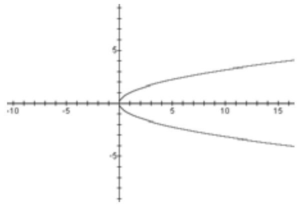
$y = \frac{1}{4p}x^2$

$V(0,0),\ F(0,p)$



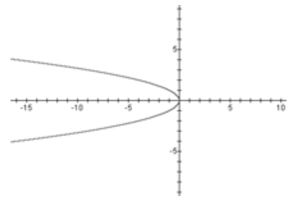
$y = -\frac{1}{4p}x^2$

$V(0,0), F(0,-p)$



$x = \frac{1}{4p}y^2$

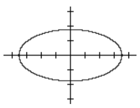
$V(0,0), F(p,0)$



$x = -\frac{1}{4p}y^2$

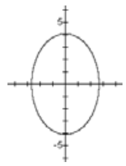
$V(0,0), F(-p,0)$

Ellipses



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$V(\pm a,0),\ C(0,0),\ F(\pm c,0)$



$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

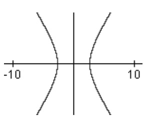
$V(0,\pm a),\ C(0,0),\ F(0,\pm c)$

$c^2 = a^2 - b^2,\ e = c/a$

major axis length $2a$

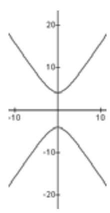
minor axis length $2b$

Hyperbolas



$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$V(\pm a,0),\ C(0,0),\ F(\pm c,0)$



$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$V(0,\pm a),\ C(0,0),\ F(0,\pm c)$

$c^2 = a^2 + b^2$

transverse axis length $2a$

conjugate axis length $2b$

Translated Conics $C(h,k)$

Parabolas $\quad y = \pm\frac{1}{4p}(x-h)^2 + k;\ x = \pm\frac{1}{4p}(y-k)^2 + h$

Ellipses $\quad \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1;\ \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

Hyperbolas $\quad \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1;\ \frac{(y-k)^2}{b^2} + \frac{(x-h)^2}{a^2} = 1$

Rotation of Axes $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$B^2 - 4AC < 0$ ellipse; $B^2 - 4AC = 0$ parabola; $B^2 - 4AC > 0$ hyperbola

$$\cot 2\phi = \frac{A - C}{B},\ 0^\circ < 2\phi < 180^\circ$$

$$x = x'\cos\phi - y'\sin\phi$$
$$y = x'\sin\phi + y'\cos\phi$$

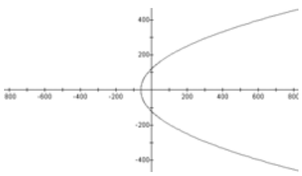
Polar Equations of Conics

Focus at the pole

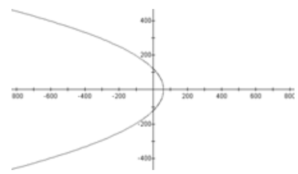
$e = 1$ parabola

$e < 1$ ellipse

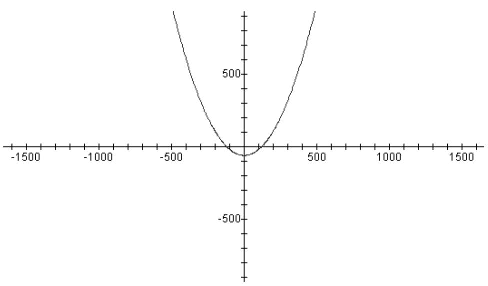
$e > 1$ hyperbola



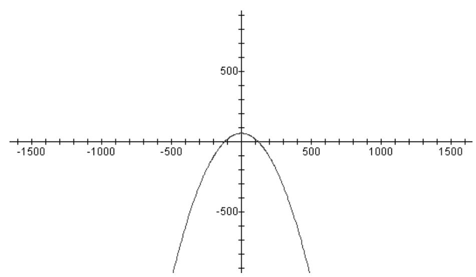
$$r = \frac{de}{1 - e\cos\theta}$$



$$r = \frac{de}{1 + e\cos\theta}$$



$$r = \frac{de}{1 - e\sin\theta}$$



$$r = \frac{de}{1 + e\sin\theta}$$

Parametric Equations

$x = f(t),\ y = g(t);\ a \le t \le b$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \qquad \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}$$

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ length of a curve that does not intersect itself

$S = \int_{t=a}^{t=b} 2\pi y\, ds = \int_{t=a}^{t=b} 2\pi g(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ area of surface of revolution about *x*-axis

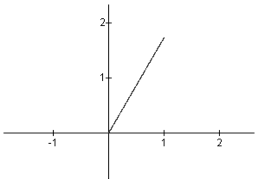
$S = \int_{t=a}^{t=b} 2\pi x\, ds = \int_{t=a}^{t=b} 2\pi f(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ area of surface of revolution about *y*-axis

Polar Coordinates (r, θ)

$$x = r\cos\theta,\ y = r\sin\theta,\ x^2 + y^2 = r^2;\ \tan\theta = \frac{y}{x}$$

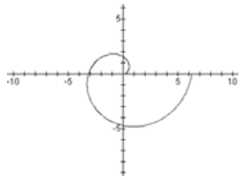
Polar Graphs $a > 0, b > 0, p > 0$

Line



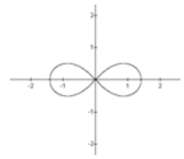
$\theta = k$

Spiral

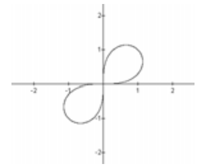


$r = \theta$

Lemniscates

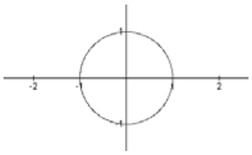


$r^2 = a\cos 2\theta$

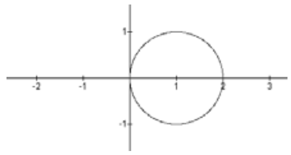


$r^2 = a\sin 2\theta$

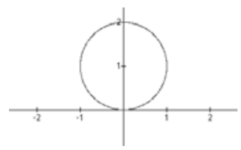
Circles



$r = a$

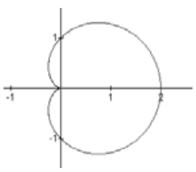


$r = 2a\cos\theta$

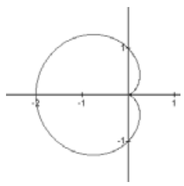


$r = 2a\sin\theta$

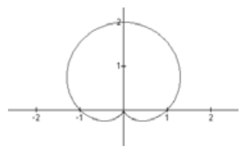
Cardioids



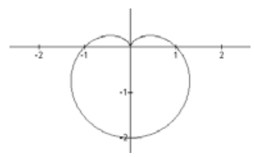
$r = a + a\cos\theta$



$r = a - a\cos\theta$

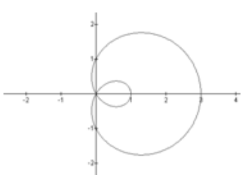


$r = a + a\sin\theta$

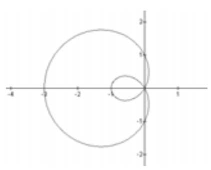


$r = a - a\sin\theta$

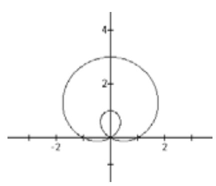
Limacons with Loop $a < b$



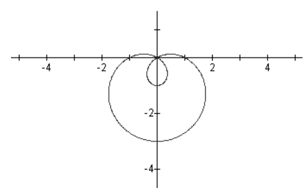
$r = a + b\cos\theta$



$r = a - b\cos\theta$

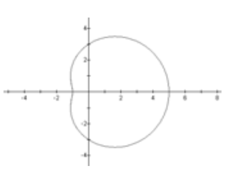


$r = a + b\sin\theta$

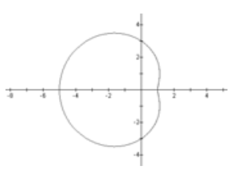


$r = a - b\sin\theta$

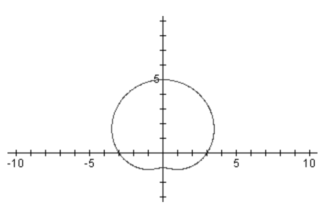
Limacons with no Loop $a > b$



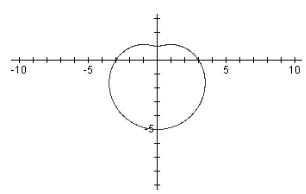
$r = a + b\cos\theta$



$r = a - b\cos\theta$

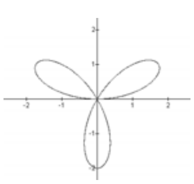


$r = a + b\sin\theta$

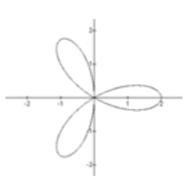


$r = a - b\sin\theta$

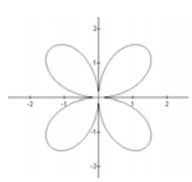
Roses



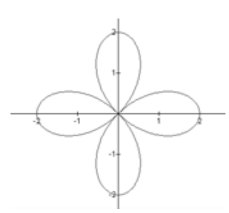
$r = a\sin 3\theta$



$r = a\cos 3\theta$



$r = a\sin 2\theta$



$r = a\cos 2\theta$

$$r = f(\theta)$$

$$m = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$A = \int_{\alpha}^{\beta} \tfrac{1}{2}\left[f(\theta)\right]^2 d\theta$$

$$A = \int_{\alpha}^{\beta} \tfrac{1}{2}\left[f(\theta)\right]^2 d\theta - \int_{\alpha}^{\beta} \tfrac{1}{2}\left[g(\theta)\right]^2 d\theta$$

$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ length of polar cuve

$S = \int_{\theta=\alpha}^{\theta=\beta} 2\pi y ds = \int_{\theta=\alpha}^{\theta=\beta} 2\pi r \sin\theta ds$ area of surface of revolution about the polar axis

$S = \int_{\theta=\alpha}^{\theta=\beta} 2\pi x ds = \int_{\theta=\alpha}^{\theta=\beta} 2\pi r \cos\theta ds$ area of surface of revolution about the line $\theta = \frac{\pi}{2}$

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$