Math 266 Fourth Exam Practice Problems

Determine if the series is absolutely convergent, conditionally convergent, or divergent.

1. $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{e^n}$ **2.** $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\ln n}$ **3.** $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ **4.** $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n)!}{n^5}$

5. Find the sum of the infinite series $\sum_{n=1}^{\infty} (-1)^n \frac{2}{\left(3^n\right)^3}$ accurate to three decimal places.

6. Find an upper bound for the error if the sum of the first four terms of the infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}$$

is used as an approximation to the sum of the series.

Determine the interval of convergence of the power series.

7. $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ **9.** $\sum_{n=1}^{\infty} \frac{(x+1)^n}{n}$ **11.** $\sum_{n=1}^{\infty} \frac{(x+1)^n}{2^n n^2}$

8. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}$ **10.** $\sum_{n=0}^{\infty} n^2 (x-1)^n$ **12.** $\sum_{n=0}^{\infty} \frac{n x^n}{\sqrt{n+1}}$

13. Find a power series representation for $\frac{1}{1-\sin x}$, then differentiate term-by-term to find a power series representation for $\frac{\cos x}{(1-\sin x)^2}$.

14. Given $f(x) = \sum_{n=0}^{\infty} \frac{(3x+1)^n}{4^n}$, find the radius of convergence of the power series and the domain of f.

Write the power series that defines the function f', find its radius of convergence, and find the domain of f'.

15. Find a power series representation of the integral $\int_0^x \frac{dt}{5+t}$ and determine its radius of convergence.

16. Compute accurate to three decimal places the value of $\int_0^{1/4} e^{\sqrt{x}} dx$ by using series.

17. Find the first four nonzero terms of the Maclaurin series for $e^{-x} \sin x$.

18. Find a power series representation for $f(x) = \sqrt{x+1}$ at 3, and determine its radius of convergence.

19. Use a power series to compute the value of $\sin 32^\circ$ to four decimal places.

20. Compute accurate to three decimal places the value of $\int_0^{1/3} \cos x^2 \, dx$.

21. Use a binomial series to find the Maclaurin series for $f(x) = \frac{1}{\sqrt{1-x^2}}$, and determine its radius of convergence.

22. Compute the value of $1/\sqrt[3]{218}$ accurate to three decimal places by using a binomial series.

Answers

1. AC

2. D

3. CC

4. D

5. -0.071

6. $1/10!$

7. $[-1, 1]$

8. $(-1, 1)$

9. $[-2, 0)$

10. $(0, 2)$

11. $[-3, 1]$

12. $(-1, 1)$

13. $\sum_{n=0}^{\infty} \sin^n x, |x| < \pi/2; \sum_{n=0}^{\infty} (n+1) \cos x \sin^n x$, all $x \neq \frac{k\pi}{2}, k$ integer

14. $r = 4/3; (-5/3, 1); f'(x) = \sum_{n=0}^{\infty} \frac{3n(3x+1)^{n-1}}{4^n}; r = 4/3; (-5/3, 1)$

15. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)5^{n+1}}$; $r = 5$

16. 0.351

17. $x - x^2 + \frac{x^3}{3} - \frac{x^5}{30}$

18. $2 + \frac{1}{4}(x-3) + 2\sum_{n=2}^{\infty} \frac{(-1)^{n+1} 1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-3)(x-3)^n}{2^{3n} n!}; r = 4$

19. 0.5299

20. 0.333

21. $1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{2^n n!} x^{2n}$; $r = 1$

22. 0.166