Math 266 Third Exam Practice Problems

Find the limits.

1. $\lim_{x \to 0} \dfrac{8x^2}{\cos x - 1}$

2. $\lim_{x \to 1} \dfrac{x - 1}{\ln x - \sin \pi x}$

3. $\lim_{x \to \frac{\pi}{2}^-} \left(\dfrac{\pi}{2} - x \right) \tan x$

4. $\lim_{\theta \to 0} \dfrac{(1/2)^{\theta} - 1}{\theta}$

5. $\lim_{x \to \infty} \dfrac{\log_2 x}{\log_3 (x + 3)}$

6. $\lim_{x \to 0^+} (\ln x - \ln \sin x)$

7. $\lim_{x \to \infty} \dfrac{1}{x \ln x} \int_1^x \ln t \, dt$

8. $\lim_{x \to e^+} (\ln x)^{1/(x-e)}$

9. $\lim_{x \to \infty} x^{1/\ln x}$

10. $\lim_{x \to 0^+} \dfrac{\sqrt{x}}{\sqrt{\sin x}}$

Determine whether the improper integral is convergent or divergent. If it is convergent, evaluate it.

11. $\int_0^4 \dfrac{dx}{\sqrt{4 - x}}$

12. $\int_{-\infty}^3 \dfrac{dx}{x^2 + 9}$

13. $\int_{-1}^2 \dfrac{dx}{\sqrt[3]{x}}$

14. Determine if it is possible to assign a finite number to represent the measure of the area of the region below the curve having the equation $y = \cot^{-1} x$, to the right of the *y*-axis, and above the *x*-axis. If a finite number can be assigned, find it.

15. Determine if it is possible to assign a finite number to represent the measure of the volume of the solid formed by revolving the region to the right of the y-axis bounded by the curve having the equation $y = e^{-x}$ and the line $y = 0$ about the *x*-axis. If a finite number can be assigned, find it.

Write the first four elements of the sequence and determine whether it converges or diverges. If the sequence converges, find its limit.

16. $\left\{ \dfrac{n + 2}{3n - 1} \right\}$

17. $\left\{ \dfrac{2^n}{n + 1} \right\}$

18. $\left\{ \dfrac{\ln n}{n} \right\}$

19. $\left\{ \sqrt{n^2 + 1} - n \right\}$

Write the first four terms of the infinite series and determine whether the series is convergent or divergent. For # 20 – 23, if the series is convergent, find its sum.

20. $\sum_{n=1}^{\infty} \dfrac{2}{4^{n-1}}$ (also find a formula for S_n)

21. $\sum_{n=1}^{\infty} \dfrac{n^2 - 1}{4n^2 - 2n - 1}$

22. $\sum_{n=1}^{\infty} \left(\dfrac{1}{3} \right)^n$

23. $\sum_{n=0}^{\infty} \left(\dfrac{3}{2^n} - \dfrac{4}{5^n} \right)$

24. $\sum_{n=1}^{\infty} \dfrac{1}{2n - 1}$

25. $\sum_{n=1}^{\infty} \dfrac{n^2}{n^3 + 1}$

26. $\sum_{n=1}^{\infty} \dfrac{1}{\sqrt{n^2 + 9}}$

27. $\sum_{n=1}^{\infty} \dfrac{1}{n^2 (n + 1)}$

28. $\sum_{n=1}^{\infty} \dfrac{1}{(1 + 1/n)^n}$

29. $\sum_{n=1}^{\infty} \dfrac{1}{\cosh^2 n}$

30. $\sum_{n=1}^{\infty} \dfrac{1}{2n(n + 1)}$

Answers: Please call my attention to any obvious mistakes. Need problems for ratio and root tests.

1. -16

2. $\dfrac{1}{1+\pi}$

3. -1

4. $\ln\dfrac{1}{2}$

5. $\dfrac{\ln 3}{\ln 2}$

6. 0

7. 0

8. $e^{1/e}$

9. e

10. 1

11. 4; C

12. $\dfrac{\pi}{4}$; C

13. $\dfrac{3}{2}\left(2^{2/3}-1\right)$; C

14. D

15. $\dfrac{\pi}{2}$ cu. units; C

16. $\dfrac{1}{3}$; C

17. ∞; D

18. 0; C

19. 0; C

20. $S_n = \dfrac{8}{3}\left(1-\dfrac{1}{4^n}\right)$; C g.s.; $S = \dfrac{8}{3}$

21. $a_n \to \dfrac{1}{4} \neq 0$; D by the nth-term test

22. C g.s; $S = \dfrac{1}{2}$

23. C g.s.; $S = 1$ (note the limits)

24. D by BCT with h.s.

25. D by LCT with h.s.

26. D by LCT with h.s.

27. C by BCT with p-series (p=3)

28. $a_n \to \dfrac{1}{e} \neq 0 \Rightarrow D$ by the nth-term test

29. C by IT

30. C by BCT with p-series (p=2)