Math 266 Third Exam Review Notes

Indeterminate Forms

1. $\frac{0}{0}$ or $\frac{\infty}{\infty}$: Use L'Hopital's Rule: differentiate numerator and denominator **separately**

2. $0 \cdot \infty$: Rewrite the problem to get either of the forms in #1
Use L'Hopital's Rule

3. 1^{∞} or 0^{∞} or ∞^{0} : Take the limit of the ln of the function to get form #2, say the answer is L, then the answer to the original problem is e^{L} .

4. $\infty - \infty$: Either **a.** combine the expressions to get either form in #1, or
b. rationalize

Improper Integrals

1. $\int_a^{\infty} f(x)dx = \lim_{t\to\infty}\int_a^t f(x)dx$; $\int_{-\infty}^{b} f(x)dx = \lim_{t\to-\infty}\int_t^b f(x)dx$; $\int_{-\infty}^{\infty} f(x)dx = \lim_{t\to-\infty}\int_t^0 f(x)dx + \lim_{t\to\infty}\int_0^t f(x)dx$

2. If $f(x)$ is discontinuous at *a*, then $\int_a^b f(x)dx = \lim_{t\to a^+}\int_t^b f(x)dx$ **.**

If $f(x)$ is discontinuous at *b*, then $\int_a^b f(x)dx = \lim_{t\to b^-}\int_a^t f(x)dx$ **.**

If $f(x)$ is discontinuous at *c* between *a* and *b*, then $\int_a^b f(x)dx = \lim_{t\to c^-}\int_a^t f(x)dx + \lim_{t\to c^+}\int_t^b f(x)dx$ **.**

Infinite Sequence $\{a_n\} = a_1 + a_2 + \ldots + a_n + \ldots$

Infinite Series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \ldots + a_n + \ldots$

<u>Some important series:</u>

1. telescoping series ex. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ convergent

2. harmonic series $\sum_{n=1}^{\infty}\frac{1}{n}$ divergent

3. geometric series $\sum_{n=1}^{\infty} a r^{n-1}$ convergent for $|r| < 1$; divergent for $|r| \ge 1$

4. p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ convergent for $p > 1$; divergent for $p < 1$

<u>How to determine the convergence or divergence of a series:</u>

1. Find a formula for the nth **partial sum** S_n , if possible, and get $\lim_{n\to\infty} S_n$ **.** If this limit is a finite number, say *S*, then the series $\sum_{n=1}^{\infty} a_n$ converges and the **sum** is *S*; otherwise, the series diverges. Use this for telescoping series and geometric series.

2. Do the **nth-term test**: If $\lim_{n\to\infty} a_n \neq 0$ then the series $\sum_{n=1}^{\infty} a_n$ diverges.

If $\lim_{n\to\infty} a_n = 0$, no conclusion as to convergence or divergence of $\sum_{n=1}^{\infty} a_n$.

Use another test.

For series with **positive terms**:

3. integral test

$\sum_{n=1}^{\infty} a_n$ converges if $\int_1^{\infty} f(x)dx$ converges; $\sum_{n=1}^{\infty} a_n$ diverges if $\int_1^{\infty} f(x)dx$ diverges

where f should satisfy 4 conditions:

(i) $f(n) = a_n$; (ii) f is positive-valued, (iii) continuous, and (iv) decreasing for $x \geq 1$

4. basic comparison test

$a_n \leq b_n$ for every positive integer n

(i) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ also converges.

(ii) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ also diverges.

5. limit comparison test

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then either both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge or both diverge.

To find a suitable $\sum_{n=1}^{\infty} b_n$ to use when a_n is a quotient, *delete* all terms in the numerator and denominator of a_n except those that have the greatest effect on magnitude.

6. ratio test

Suppose $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$

(i) If $L < 1$, $\sum_{n=1}^{\infty} a_n$ converges.

(ii) If $L > 1$ or $L = \infty$, $\sum_{n=1}^{\infty} a_n$ diverges.

(iii) If $L = 1$, no conclusion. Use a different test.

7. root test

Suppose $\lim_{n\to\infty} \sqrt[n]{a_n} = L$

(i) If $L < 1$, $\sum_{n=1}^{\infty} a_n$ converges.

(ii) If $L > 1$ or $L = \infty$, $\sum_{n=1}^{\infty} a_n$ diverges.

(iii) If $L = 1$, no conclusion. Use a different test.