

## Math 266 First Exam Practice Problems

Determine whether or not the given function is one-to-one and, if so, find its inverse.

1.  $f(x) = x^2 - 3x + 2$

2.  $f(x) = \frac{x+2}{x+1}$

In the following, the given function  $f$  is differentiable. Verify that  $f$  has an inverse and find

$(f^{-1})'(c)$ .

3.  $f(x) = x + \sin x$ ;  $c = 0$

4.  $f(x) = e^{3x} + 2e^x - 5$ ,  $x \geq 0$ ;  $c = -2$

Differentiate. No need to simplify the final answer.

5.  $f(x) = \ln(\ln x)$

6.  $f(x) = \sin(\ln x)$

7.  $f(x) = \ln(\cos x)$

8.  $f(x) = \ln \sqrt{\frac{1-x}{2-x}}$

9.  $f(x) = e^{\sin 2x}$

10.  $y = e^{\sqrt{x}} \ln \sqrt{x}$

11.  $y = \ln(\cos e^{2x})$

12.  $f(x) = \sec(e^{\tan x})$

13.  $y = (e^{2x} - e^{-2x})^2$

14.  $g(x) = \log_5 \left( \frac{x}{x+1} \right)$

15.  $y = 2^{5x} 3^{\ln x}$

16.  $y = (x^2 + 1)^{2x}$

17.  $y = (\sin x)^{\cos x}$

18.  $f(x) = \tan^{-1}(e^x)$

19.  $f(x) = x \sin^{-1} 2x$

20.  $y = \sin[\sec^{-1}(\ln x)]$

21.  $g(x) = \ln(\tan^{-1} 3x)$

22.  $y = (\sinh x)^x$

23.  $f(x) = \cosh(\ln x^3)$

24.  $y = \frac{\sinh x}{\cosh x - 1}$

Use logarithmic differentiation to find  $y'$ .

25.  $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$

26.  $y = \sqrt[3]{(3x-1)\sqrt{2x+5}}$

Evaluate the integral.

27.  $\int \frac{\sec^2 2x}{4 - \tan 2x} dx$

28.  $\int \frac{dx}{\sqrt{x}(1 + \sqrt{x})}$

29.  $\int \frac{dx}{x(\ln x)^2}$

30.  $\int \frac{2e^x}{\sqrt[3]{e^x + 1}} dx$

31.  $\int \frac{\sin(e^{-2x})}{e^{2x}} dx$

32.  $\int e^{-x} [1 + \cos(e^{-x})] dx$

33.  $\int x 10^{-x^2} dx$

34.  $\int \frac{\log_3 \sqrt{x} - 1}{x} dx$

35.  $\int \frac{\tan(\ln x)}{x} dx$

36.  $\int (3 - \csc x)^2 dx$

$$37. \int \frac{x}{\sqrt{1-x^4}} dx$$

$$38. \int \frac{\sec^2 x}{\sqrt{9 - \tan^2 x}} dx$$

$$39. \int \frac{\sinh x}{1 + \cosh x} dx$$

$$40. \int \frac{1 + \tanh x}{\cosh^2 x} dx$$

Evaluate the definite integral.

$$41. \int_1^e \frac{\ln x}{x} dx$$

$$42. \int_{\ln 2}^{\ln 3} (e^x - e^{-x})^2 dx$$

$$43. \int_0^1 (2^x + x^2) dx$$

$$44. \int_{\ln 2}^{\ln 3} \frac{e^{-x}}{\sqrt{1 - e^{-2x}}} dx$$

Solve.

45. In a bacteria growing experiment, a biologist observes that the number of bacteria in a certain culture triples every four hours. After 12 hours, it is estimated that there are 1 million bacteria in the culture.
- How many bacteria were present initially?
  - What is the doubling time for the bacteria population?
46. What is the half-life of a radioactive substance if it takes 5 years for a third of the substance to decay?
47. A cup of coffee is served to you at 185° F in a room where the temperature is 65° F. Two minutes later the temperature of the coffee has dropped to 155° F. How many more minutes would you expect to wait for the coffee to cool to 105° F?
48. A particle is traveling upward and to the right along the curve  $y = \ln x$ . Its  $x$ -coordinate is increasing at the rate of
- $$dx/dt = \sqrt{x} \text{ m/sec.}$$
- At what rate is the  $y$ -coordinate changing at the point  $(e^2, 2)$ ?
49. Find the area of the “triangular” region in the first quadrant that is bounded above by the curve  $y = e^{2x}$ , below by the curve  $y = e^x$ , and on the right by the line  $x = \ln 3$ .
50. Given  $f(x) = x^2 e^{-x}$ ,
- find the domain.
  - find the intervals where the function increases/decreases
  - find the local extrema, if any
  - determine the concavity, and find the points of inflection
  - sketch the graph, indicating all asymptotes

Also review implicit differentiation.