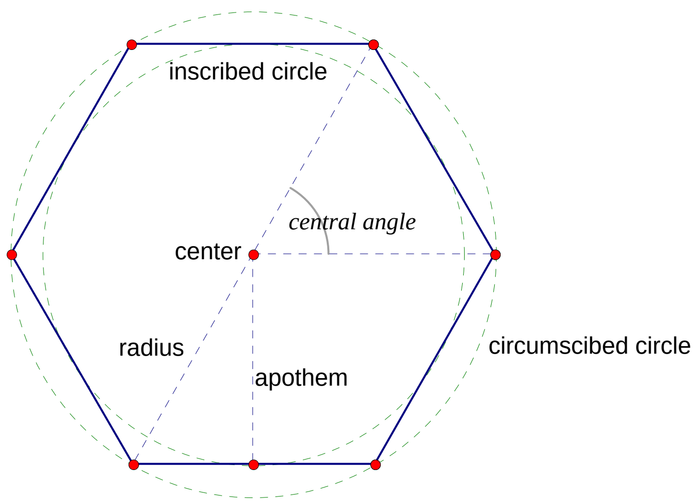
Chapter 7 Notes: Areas of Polygons and Circles

IMPORTANT TERMS AND DEFINITIONS

A ***perimeter*** of a polygon is the sum of the lengths of all sides of the polygon.

The ***center of a regular polygon*** is the common center for the inscribed and circumscribed circles of the polygon.

A ***radius*** of a regular polygon is any line segment that joins the center of the regular polygon to one of its vertices.

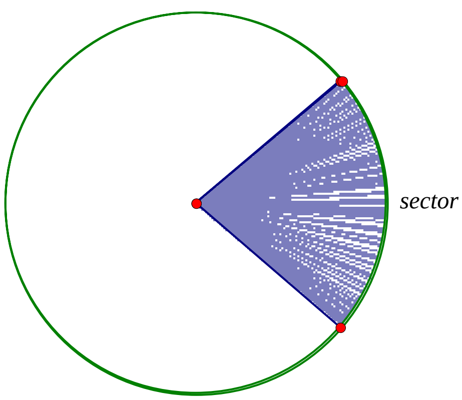


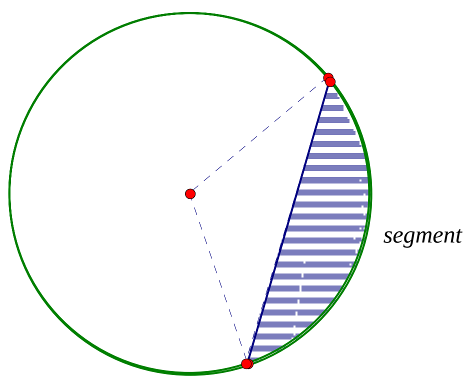
An ***apothem*** of a regular polygon is any line segment drawn from the center of that polygon perpendicular to one of the sides.

A ***central angle of a regular polygon*** is an angle formed by two consecutive radii of the regular polygon.

π is the ratio between the circumference C and the diameter d of any circle; thus $\pi = \frac{C}{d}$.

A ***sector*** of a circle is a region bounded by two radii of the circle and an arc intercepted by those radii.





A ***segment*** of a circle is a region bounded by a chord and its minor (or major) arc.

Suppose that two congruent polygons lie in parallel planes in such a way that their corresponding sides are parallel. If the corresponding vertices of these polygons are joined by line segments, then the solid that results is a ***prism***. The congruent figures that lie in parallel planes are the ***bases*** of the prism.

A ***right prism*** is a prism in which the lateral edges are perpendicular to the base edges at their points of intersection. An ***oblique prism*** is a prism in which the parallel lateral edges are oblique to the base edges at their points of intersection.

The ***lateral area*** L of a prism is the sum of the areas of all lateral faces.

For any prism, the ***total area*** T is the sum of the lateral area and the areas of the bases.

A ***regular prism*** is a right prism whose bases are regular polygons.

A ***cube*** is a right square prism, whose edges are congruent.

A ***regular pyramid*** is a pyramid whose base is a regular polygon and whose lateral edges are all congruent.

The ***slant height*** of a regular pyramid is the altitude from the vertex of the pyramid to the base of any of the congruent lateral faces.

POSTULATES:

18. (**Area Postulate**) Corresponding to every bounded region is a unique positive number A, known as the area of that region.
19. If two enclosed plane figures are congruent, then their areas are equal.
20. (**Area-Addition Postulate**) Let R and S be two enclosed regions that do not overlap. Then $A_{R\cup S} = A_R + A_S$.
21. The area A of a rectangle whose base has length b and whose altitude has length h is given by $A = bh$.
22. The ratio of the circumference of a circle to the length of its diameter is a unique positive constant.
23. The ratio of the degree measure m of the arc (or central angle) of a sector to 360° is the same as the ratio of the area of the sector to the area of the circle; that is, $\frac{\text{area of sector}}{\text{area of circle}} = \frac{m}{360^\circ}$.
24. (**Volume Postulate**) Corresponding to every solid is a unique positive number V known as the volume of the solid.
25. The volume of a right rectangular prism is given by $V = lwh$ where l measures the length, w the width, and h the altitude of the prism.
26. The volume of a right prism is given by $V = Bh$ where B is the area of a base and h is the length of the altitude of the prism.

THEOREMS AND COROLLARIES:

8.1.1 The area A of a square whose sides are each of length s is given by $A = s^2$.

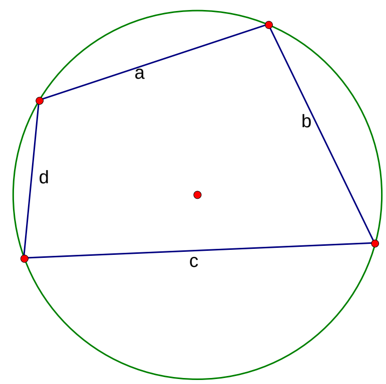
8.1.2 The area A of a parallelogram with a base of length b and with corresponding altitude of length h is given by $A = bh$.

8.1.3 The area A of a triangle whose base has length b and whose corresponding altitude has length h is given by $A = \frac{1}{2}bh$.

8.1.4 The area of a right triangle with legs of lengths a and b is given by $A = \frac{1}{2}ab$

8.2.1 (Heron's Formula) If the three sides of a triangle have lengths a, b, and c, then the area A of the triangle is given by $A = \sqrt{s(s-a)(s-b)(s-c)}$ where the semiperimeter of the triangle is $s = \frac{1}{2}(a+b+c)$.

8.2.2 (Brahmagupta's Formula) For a cyclic quadrilateral with sides of lengths $a,\ b,\ c,$ and d is given by $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$ where $s = \frac{1}{2}(a+b+c+d)$



8.2.3 The area A of a trapezoid whose bases has lengths b_1 and b_2 and whose altitude has length h is given by $A = \frac{1}{2}h(b_1 + b_2)$.

8.2.4 The area A of any quadrilateral with perpendicular diagonals of lengths d_1 and d_2 is given by $A = \frac{1}{2}d_1 d_2$.

8.2.5 The area A of a rhombus whose diagonals have lengths d_1 and d_2 is given by $A = \frac{1}{2}d_1 d_2$.

8.2.6 The area A of a kite whose diagonals have lengths d_1 and d_2 is given by $A = \frac{1}{2}d_1 d_2$.

8..2.7 The ratio of the areas of two similar triangles equals the square of the ratio of the lengths of any two corresponding sides; that is $\frac{A_1}{A_2} = \left(\frac{a_1}{a_2}\right)^2$.

8.3.1 The area A of a regular polygon whose apothem has length a and whose perimeter is P is given by $A = \frac{1}{2}aP$.

8.4.1 The circumference of a circle is given by the formula $C = \pi d$ or $C = 2\pi r$.

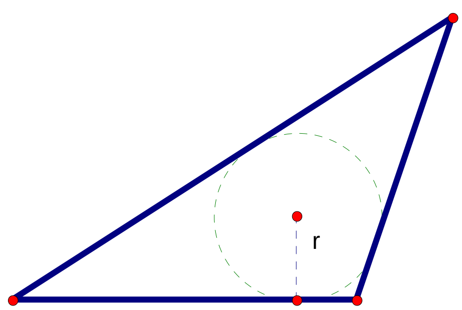
8.4.2 In a circle whose circumference is C, the length ℓ of an arc whose degree measure is m is given by $\ell = \frac{m}{360} \cdot C$. (For arc *AB*, $l\widehat{AB} = \frac{m\widehat{AB}}{360} \cdot C$)

8.4.3 The area A of a circle whose radius is of length r is given by $A = \pi r^2$.

8.5.1 In a circle of radius r, the area A of a sector whose arc has degree measure m is given by $A = \frac{m}{360}\pi r^2$.

8.5.2 The area of a semicircular region of radius r is $A = \frac{1}{2}\pi r^2$.

8.5.3 Where P represents the perimeter of a triangle and r represents the length of the radius of its inscribed circle, the area of the triangle is given by $A = \frac{1}{2}rP$.



9.1.1 The lateral area L of any prism whose altitude has measure h and whose base has perimeter P is given by $L = hP$

9.1.2 The total area T of any prism with lateral area L and base area B is given by $T = L + 2B$.

9.2.1 In a regular pyramid, the length a of the apothem of the base, the altitude h, and the slant height l satisfy the Pythagorean Theorem: that is, $l^2 = a^2 + h^2$ in every regular pyramid.

9.2.2 The lateral area L of a regular pyramid with slant height of length l and perimeter P of the base is given by $L = \frac{1}{2}lP$.

9.2.3 The total area (surface area) T of a pyramid with lateral area L and base area *B* is given by $T = L + B$.

9.2.4 The volume V of a pyramid having base area B and an altitude of length h is given by $V = \frac{1}{3}Bh$.

9.2.5 In a regular pyramid, the lengths of altitude h, radius r of the base, and lateral edge e satisfy the Pythagorean Theorem: that is, $e^2 = h^2 + r^2$.

9.3.1 The lateral area L of a right circular cylinder with altitude of length h and circumference C of the base is given by $L = hC$ or $L = 2\pi rh$.

9.3.2 The total area T of a right circular cylinder with base area B and lateral area L is given by $T = L + 2B$ or $T = 2\pi rh + 2\pi r^2$.

9.3.3 The volume V of a right circular cylinder with base area B and altitude of length h is given by $V = Bh$ or $V = \pi r^2 h$.

9.3.4 The lateral area L of a right circular cone with slant height of length l and circumference C of the base is given by $L = \frac{1}{2} lC$ or $L = \pi rl$.

9.3.5 The total area T of a right circular cone with base area B and lateral area L is given by $T = B + L$ or $T = \pi r^2 + \pi rl$.

9.3.6 In a right circular cone, the lengths of the radius r (of the base), the altitude h, and the slant height l satisfy the Pythagorean Theorem: that is, $l^2 = r^2 + h^2$ in every right circular cone.

9.3.7 The volume V of a right circular cone with base area B and altitude of length h is given by $V = \frac{1}{3} Bh$ or $V = \frac{1}{3} \pi r^2 h$.