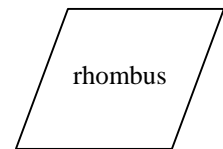
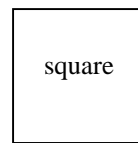
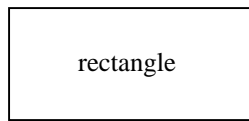
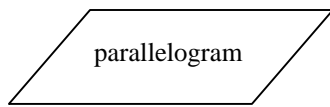


Chapters 4 and 5 Notes: Quadrilaterals and Similar Triangles

IMPORTANT TERMS AND DEFINITIONS



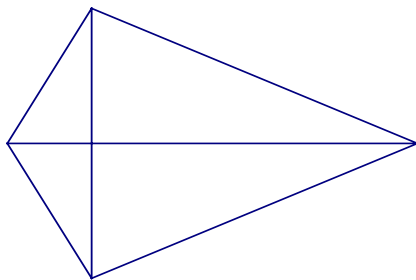
A **quadrilateral** is a polygon that has four sides.

A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.

The **distance between two parallel lines** refers to the length of the perpendicular segment between the two parallel lines.

An **altitude** of a parallelogram is a line segment from one vertex that is perpendicular to a nonadjacent side (or to an extension of that side).

A **kite** is a quadrilateral with two distinct pairs of congruent adjacent sides.



Kite shown with diagonals

A **rectangle** is a parallelogram that has a right angle.

A **square** is a rectangle that has two congruent adjacent sides.

A **rhombus** is a parallelogram with two congruent adjacent sides.

A **trapezoid** is a quadrilateral with exactly two parallel sides.

In a trapezoid, the nonparallel sides are called the **legs** and the parallel sides are called the **bases**. The pair of angles that have a common base for a side are called **base angles**. Base angles exist in pairs.

The **median** of a trapezoid is a line segment that joins the midpoints of the two legs.

If the two legs of a trapezoid are congruent, the trapezoid is known as an **isosceles trapezoid**.

An **altitude** of a trapezoid is a line segment from one vertex of one base of the trapezoid perpendicular to the opposite base (or an extension of that base).

A **cyclic quadrilateral** is a quadrilateral such that all its vertices lie on a circle. For a quadrilateral to be cyclic, the perpendicular bisectors of the sides must intersect at a point that is equidistant from all four vertices (e.g. squares, rectangles, isosceles trapezoid).

A **ratio** is the quotient $\frac{a}{b}$ (where $b \neq 0$) that provides a comparison between numbers a and b . If units of measure are involved, these units must be **commensurable** (convertible to the same unit of measure). If two quantities cannot be compared because no common unit of measure is possible, the quantities are **incommensurable**.

A **rate** is a quotient that compares two quantities that are incommensurable.

A **proportion** is a statement that two ratios or two rates are equal. The proportion $\frac{a}{b} = \frac{c}{d}$ may be read as “ a is to b as c is to d ”, where a and d are called the **extremes**, while b and c are called the **means**.

In $\frac{a}{b} = \frac{b}{c}$, where the means of the proportion are identical, the value of b is known as the **geometric mean** of a and c .

An **extended ratio** compares more than two quantities and must be expressed in a form such as $a:b:c$ or $d:e:f:g$.

Two polygons are **similar** if and only if two conditions are satisfied:

1. All pairs of corresponding angles are congruent.
2. All pairs of corresponding sides are proportional.

Two congruent polygons are also similar polygons.

CSSTP: Corresponding sides of similar triangles are proportional.

CASTC: Corresponding angles of similar triangles are congruent.

A **Pythagorean triple** is a set of three natural numbers (a, b, c) for which $a^2 + b^2 = c^2$.

Properties of Proportions:

1. (**Means-Extremes Property**) In a proportion, the product of the means equals the product of the extremes; that is if $\frac{a}{b} = \frac{c}{d}$ (where $b \neq 0$ and $d \neq 0$), then $a \cdot d = b \cdot c$.
2. In a proportion, the means or the extremes (or both the means and the extremes) may be interchanged; that is, if $\frac{a}{b} = \frac{c}{d}$ (where $a, b, c,$ and d are nonzero), then $\frac{a}{c} = \frac{b}{d}$, $\frac{d}{b} = \frac{c}{a}$, and $\frac{d}{c} = \frac{b}{a}$.
3. If $\frac{a}{b} = \frac{c}{d}$ (where $b \neq 0$ and $d \neq 0$), then $\frac{a+b}{b} = \frac{c+d}{d}$ and $\frac{a-b}{b} = \frac{c-d}{d}$.
4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+c}{b+d} = \frac{a}{b} = \frac{c}{d}$.

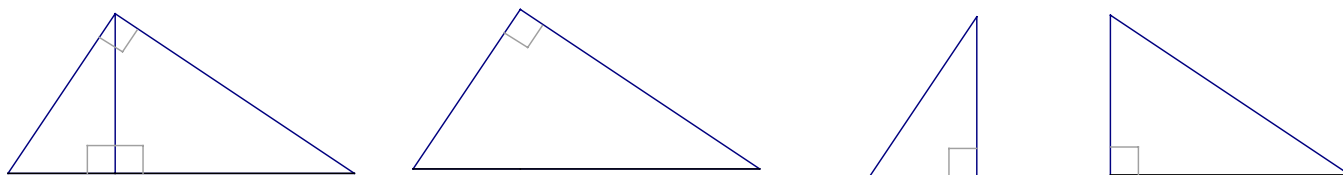
POSTULATE:

15. (AAA) If the three angles of one triangle are congruent to the three angles of a second triangle, then the triangles are similar.

THEOREMS AND COROLLARIES:

- 4.1.1 A diagonal of a parallelogram separates it into two congruent triangles.
- 4.1.2 Opposite angles of a parallelogram are congruent.
- 4.1.3 Opposite sides of a parallelogram are congruent.
- 4.1.4 Diagonals of a parallelogram bisect each other.
- 4.1.5 Consecutive angles of a parallelogram are supplementary.
- 4.1.6 Two parallel lines are everywhere equidistant.
- 4.1.7 If two sides of one triangle are congruent to two sides of a second triangle and the included angle of the first triangle is greater than the included angle of the second triangle, then the length of the side opposite the included angle of the first triangle is greater than the length of the side opposite the included angle of the second.
- 4.1.8 In a parallelogram with unequal pairs of consecutive angles, the longer diagonal lies opposite the obtuse angle.
- 4.2.1 If two sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.
- 4.2.2 If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram.
- 4.2.3 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- 4.2.4 In a kite, one pair of opposite angles are congruent.
- 4.2.5 The segment that joins the midpoints of two sides of a triangle is parallel to the third side and has a length equal to one-half the length of the third side.
- 4.3.1 All angles of a rectangle are right angles.
- 4.3.2 The diagonals of a rectangle are congruent.
- 4.3.3 All sides of a square are congruent.
- 4.3.4 All sides of a rhombus are congruent.
- 4.3.5 The diagonals of a rhombus are perpendicular.
- 4.4.1 The base angles of an isosceles trapezoid are congruent.
- 4.4.2 The diagonals of an isosceles trapezoid are congruent.
- 4.4.3 The length of the median of a trapezoid equals one-half the sum of the lengths of the two bases.

- 4.4.4 The median of a trapezoid is parallel to each base.
- 4.4.5 If two base angles of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.
- 4.4.6 If the diagonals of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.
- 4.4.7 If three (or more) parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on any transversal.
- 5.3.1 **(AA)** If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.
- 5.3.2 The lengths of the corresponding altitudes of similar triangles have the same ratio as the lengths of any pair of corresponding sides.
- 5.3.3 **(SAS~)** If an angle of one triangle is congruent to an angle of a second triangle and the pairs of sides including these angles are proportional, then the triangles are similar.
- 5.3.4 **(SSS~)** If the three sides of one triangle are proportional (in length) to the three sides of a second triangle, then the triangles are similar.
- 5.3.5 If a line segment divides two sides of a triangle proportionally, then this line segment is parallel to the third side of the triangle.



- 5.4.1 The altitude drawn to the hypotenuse of a right triangle separates the right triangle into two right triangles that are similar to each other and to the original right triangle.
- 5.4.2 The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.
- 5.4.3 The length of each leg of a right triangle is the geometric mean of the hypotenuse and the length of the segment of the hypotenuse adjacent to that leg.
- 5.4.4 **(Pythagorean Theorem)** The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs.
- 5.4.5 **(Converse of Pythagorean Theorem)** If a , b , and c are the lengths of the three sides of a triangle, with c the length of the longest side, and if $c^2 = a^2 + b^2$, then the triangle is a right triangle with the right angle opposite the side of length c .
- 5.4.6 **(HL)** If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent.

5.4.7 Let a , b , and c represent the lengths of the three sides of a triangle with c the length of the longest side.

1. If $c^2 > a^2 + b^2$, then the triangle is obtuse and the obtuse angle lies opposite the side of length c .
2. If $c^2 < a^2 + b^2$, then the triangle is acute.

5.5.1 **(45-45-90 Theorem)** In a triangle, whose angles measure 45° , 45° , and 90° , the hypotenuse has a length equal to the product of $\sqrt{2}$ and the length of either leg.

5.5.2 **(30-60-90 Theorem)** In a triangle whose angles measure 30° , 60° , and 90° , the hypotenuse has a length equal to twice the length of the shorter leg, and the length of the longer leg is the product of $\sqrt{3}$ and the length of the shorter leg.

5.5.3 If the length of the hypotenuse of a right triangle equals the product of $\sqrt{2}$ and the length of either leg, then the angles of the triangle measure 45° , 45° , and 90° .

5.5.4 If the length of the hypotenuse of a right triangle is twice the length of one leg of the triangle, then the angle of the triangle opposite that leg measures 30° .

5.6.1 If a line is parallel to one side of a triangle and intersects the other two sides, then it divides these sides proportionally.

5.6.2 When three (or more) parallel lines are cut by a pair of transversals, the transversals are divided proportionally by the parallel lines.

5.6.3 **(The Angle-Bisector Theorem)** If a ray bisects one angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the two sides that form the bisected angle.

5.6.4 **(Ceva's Theorem)** Let D be any point in the interior of $\triangle ABC$, and let \overline{BE} , \overline{AF} , and \overline{CG} be the line segments determined by D and vertices of $\triangle ABC$. Then the product of the ratios of the lengths of the segments of each of the three sides (taken in order from a given vertex of the triangle) equals 1; that is, $\frac{AG}{GB} \cdot \frac{BF}{FC} \cdot \frac{CE}{EA} = 1$.

