

Chapters 2 and 3 Notes: Parallel Lines and Triangles

CONSTRUCTIONS:

- To construct the line that is perpendicular to a given line from a point not on the given line.
- To construct the line parallel to a given line from a point not on that line.

POSTULATES:

- (**Parallel Postulate**) Through a point not on a line, exactly one line is parallel to the given line.
- If two parallel lines are cut by a transversal, then the corresponding angles are congruent.
- (**SSS**) If the three sides of one triangle are congruent to the three sides of a second triangle, then the triangles are congruent.
- (**SAS**) If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.
- (**ASA**) If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the triangles are congruent.

THEOREMS, COROLLARIES AND LEMMAS:

- 2.1.1 From a point not on the given line, there is exactly one line perpendicular to the given line.
- 2.1.2 If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.
- 2.1.3 If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.
- 2.1.4 If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.
- 2.1.5 If two parallel lines are cut by a transversal, then the exterior angles on the same side of the transversal are supplementary.
- 2.3.1 If two lines are cut by a transversal so that the corresponding angles are congruent, then these lines are parallel.
- 2.3.2 If two lines are cut by a transversal so that the alternate interior angles are congruent, then these lines are parallel.
- 2.3.3 If two lines are cut by a transversal so that the alternate exterior angles are congruent, then these lines are parallel.
- 2.3.4 If two lines are cut by a transversal so that the interior angles on the same side of the transversal are supplementary, then these lines are parallel.
- 2.3.5 If two lines are cut by a transversal so that the exterior angles on the same side of the transversal are supplementary, then these lines are parallel.
- 2.3.6 If two lines are both parallel to a third line, then these lines are parallel to each other.
- 2.3.7 If two coplanar lines are both perpendicular to a third line, then these lines are parallel to each other.
- 2.4.1 In a triangle, the sum of the measures of the interior angles is 180° .
- 2.4.2 Each angle of an equiangular triangle measures 60° .
- 2.4.3 The acute angles of a right triangle are complementary.
- 2.4.4 If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.
- 2.4.5 The measure of an exterior angle of a triangle equals the sum of the measures of the two non-adjacent interior angles.
- 2.5.1 The total number of diagonals D in a polygon of n sides is given by the formula $D = \frac{n(n-3)}{2}$.
- 2.5.2 The sum S of the measures of the interior angles of a polygon with n sides is given by the formula $S = (n-2) \cdot 180^\circ$.
- 2.5.3 The measure I of each interior angle of a regular polygon of n sides is $I = \frac{(n-2) \cdot 180^\circ}{n}$.
- 2.5.4 The sum of the measures of the four interior angles of a quadrilateral is 360° .
- 2.5.5 The sum of the measures of the exterior angles, one at each vertex, of a polygon is 360° .

2.5.6 The measure E of each exterior angle of a regular polygon of n sides is $E = \frac{360^\circ}{n}$.

3.1.1 (AAS) If two angles and a non-included side of one triangle are congruent to angles and a non-included side of a second triangle, then the triangles are congruent.

3.2.1 (HL) If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent.

3.3.1 Corresponding altitudes of congruent triangles are congruent.

3.3.2 The bisector of the vertex angle of an isosceles triangle separates the triangle into two congruent triangles.

3.3.3 If two sides of a triangle are congruent, then the angles opposite these sides are also congruent.

3.3.4 If two angles of a triangle are congruent, then the sides opposite these angles are also congruent.

3.3.5 An equilateral triangle is also equiangular.

3.3.6 An equiangular triangle is also equilateral.

3.5.1 The measure of a line segment is greater than the measure of any of its parts.

3.5.2 The measure of an angle is greater than the measure of any of its parts.

3.5.3 The measure of an exterior angle of a triangle is greater than the measure of either non-adjacent interior angle.

3.5.4 If a triangle contains a right or an obtuse angle, then the measure of this angle is greater than the measure of either of the remaining angles.

3.5.5 (Addition Property of Inequality) If $a > b$ and $c > d$, then $a + c > b + d$.

3.5.6 If one side of a triangle is longer than a second side, then the measure of the angle opposite the first side is greater than the measure of the angle opposite the second side.

3.5.7 If the measure of one angle of a triangle is greater than the measure of a second angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

3.5.8 The perpendicular segment from a point to a line is the shortest segment that can be drawn from the point to the line.

3.5.9 The perpendicular segment from a point to a plane is the shortest segment that can be drawn from the point to the plane.

3.5.10 (Triangle Inequality) The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

3.5.10 (Alternative) The length of one side of a triangle must be between the sum and the difference of the lengths of the other two sides.

Pythagorean Theorem The square of the length (c) of the hypotenuse of a right triangle equals the sum of the squares of the lengths (a and b) of the legs of the right triangle; i.e., $c^2 = a^2 + b^2$.

Square Root Property Let x represent the length of a line segment, and let p represent a positive number. If

$$x^2 = p, \text{ then } x = \sqrt{p}.$$

IMPORTANT TERMS AND DEFINITIONS

Parallel lines are lines in the same plane that do not intersect.

A **transversal** is a line that intersects two or more other lines at distinct points; all of the lines lie in the same plane.

Corresponding angles $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$, $\angle 1$ and $\angle 5$

Interior angles $\angle 3, \angle 4, \angle 5, \angle 6$

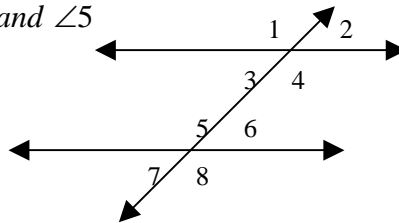
Exterior angles $\angle 1, \angle 2, \angle 7, \angle 8$

Alternate interior angles $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$

Alternate exterior angles $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$

ITPLACBAT "If two parallel lines are cut by a transversal"

ITLACBAT "If two lines are cut by a transversal"



A **triangle** is the union of three line segments that are determined by three noncollinear points.

Triangles Classified by Congruent Sides

Scalene no congruent sides
Isosceles at least two congruent sides
Equilateral all three sides congruent

Triangles Classified by Angles

Acute all angles acute
Obtuse one obtuse angle
Right one right angle
Equiangular all angles congruent

When the sides of a triangle are extended, each angle that is formed by a side and an extension of the adjacent side is an **exterior angle** of the triangle.

A **polygon** is a closed plane figure whose sides are line segments that intersect only at the endpoints.

A **diagonal** of a polygon is a line segment that joins two nonconsecutive vertices.

A **regular polygon** is a polygon that is both equilateral and equiangular.

A **polygram** is the star-shaped figure that results when the sides of certain polygons are extended.

Two triangles are **congruent** when the six parts of the first triangle are congruent to the six corresponding parts of the second triangle.

Two sides that form an angle of a triangle are said to **include that angle** of the triangle.

Any two angles of a triangle must have a common side, and these two angles are said to **include that side**.

In a right triangle, the side opposite the right angle is the **hypotenuse** of the triangle, and the sides of the right angle are the **legs** of the triangle.

In an isosceles triangle, the two sides of equal length are **legs** and the third side is the **base**. The point at which the two legs meet is the **vertex** of the triangle, so the angle formed by the legs (and opposite the base) is called the **vertex angle**. The two remaining angles are the **base angles**.

Each angle of a triangle has a unique **angle bisector**. A **median** of a triangle is a line segment that joins a vertex to the midpoint of the opposite side. An **altitude** is a line segment drawn from a vertex to the opposite side such that it is perpendicular to the opposite side.

The **perimeter** of a triangle is the sum of the lengths of its sides.

Let a and b be real numbers. $a > b$ if and only if there is a positive number p for which $a = b + p$.

CPCTC Corresponding parts of congruent triangles are congruent.

Method of Indirect Proof

To prove the statement $P \rightarrow Q$ or to complete the proof problem of the form

Given: P

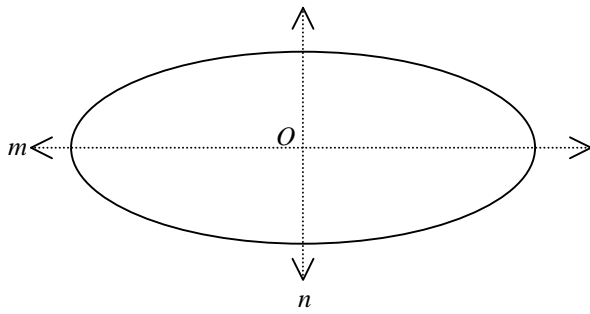
Prove: Q

where Q may be a negation, use the following steps:

1. Suppose that $\sim Q$ is true.
2. Reason from the supposition until you reach a contradiction.
3. Note that the supposition claiming that $\sim Q$ is true must be false and that Q must therefore be true.

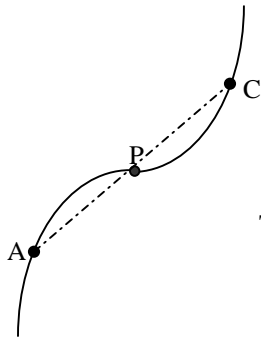
Symmetry

A figure has **symmetry with respect to a line l** if for every point A on the figure there is a second point B on the figure for which line l is the perpendicular bisector of \overline{AB} .



The oval-shaped figure shown has symmetry with respect to lines m and n . Is it symmetric with respect to point O ?

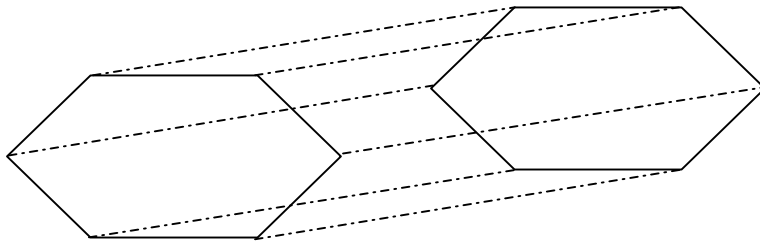
A figure has *symmetry with respect to a point P* if for every point A on the figure there is a second point C for which P is the midpoint of \overline{AC} .



The curve shown has symmetry with respect to point P.

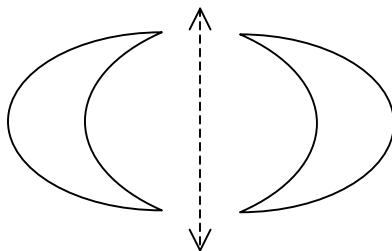
Transformations

Slides (Translations): Every point of the original figure is associated with a new point by using a fixed length and direction.



The hexagon on the left is translated to the hexagon on the right through a fixed distance and direction.

Reflections: Every point of the original figure is reflected across a line in such a way as to make the given line a line of symmetry. Each pair of corresponding points will lie on opposite sides of the line of reflection at like distances.



Rotations: Every point of the given figure leads to a point (its image) by rotation about a given point through a prescribed angle measure.

