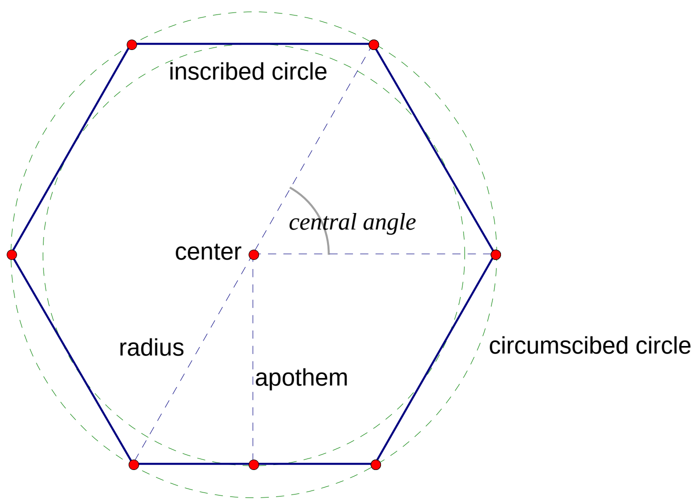
# Chapter 7 Notes:  Areas of Polygons and Circles

## IMPORTANT TERMS AND DEFINITIONS

A ***perimeter*** of a polygon is the sum of the lengths of all sides of the polygon.

The ***center of a regular polygon*** is the common center for the inscribed and circumscribed circles of the polygon.

A ***radius*** of a regular polygon is any line segment that joins the center of the regular polygon to one of its vertices.

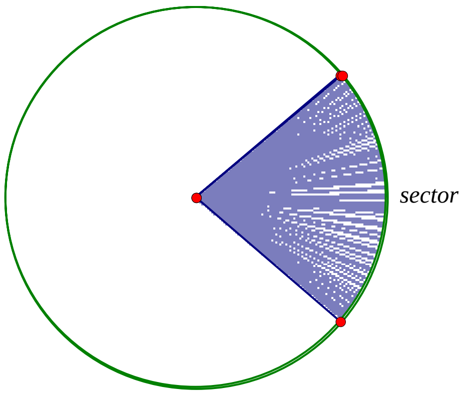


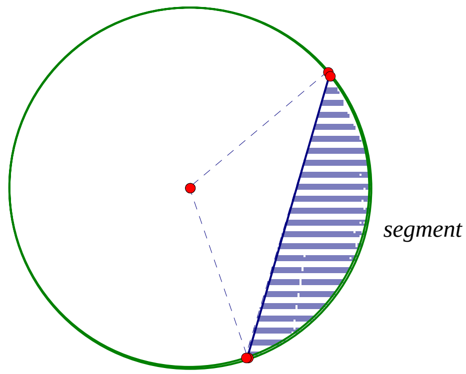

An ***apothem*** of a regular polygon is any line segment drawn from the center of that polygon perpendicular to one of the sides.

A ***central angle of a regular polygon*** is an angle formed by two consecutive radii of the regular polygon.

$\pi$ is the ratio between the circumference $C$ and the diameter $d$ of any circle; thus $\pi = \frac{C}{d}$.

A ***sector*** of a circle is a region bounded by two radii of the circle and an arc intercepted by those radii.




A ***segment*** of a circle is a region bounded by  a chord and its minor (or major) arc.

## POSTULATES:

18. (**Area Postulate**) Corresponding to every bounded region is a unique positive number $A$, known as the area of that region.
19. If two enclosed plane figures are congruent, then their areas are equal.
20. (**Area-Addition Postulate**) Let $R$ and $S$ be two enclosed regions that do not overlap. Then $A_{R \cup S} = A_R + A_S$.

**21.** The area $A$ of a rectangle whose base has length $b$ and whose altitude has length $h$ is given by $A = bh$.

**22.** The ratio of the circumference of a circle to the length of its diameter is a unique positive constant.

**23.** The ratio of the degree measure $m$ of the arc (or central angle) of a sector to $360^\circ$ is the same as the ratio of the area of the sector to the area of the circle; that is, $\frac{\text{area of sector}}{\text{area of circle}} = \frac{m}{360^\circ}$.

## THEOREMS AND COROLLARIES:

7.1.1 The area $A$ of a square whose sides are each of length $s$ is given by $A = s^2$.

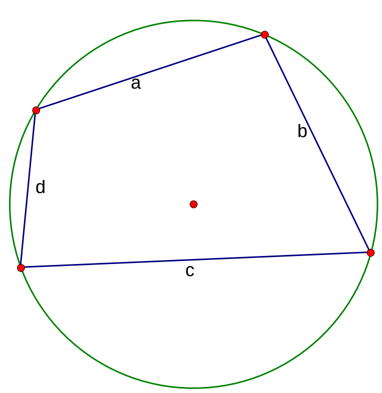
7.1.2 The area $A$ of a parallelogram with a base of length $b$ and with corresponding altitude of length $h$ is given by $A = bh$.

7.1.3 The area $A$ of a triangle whose base has length $b$ and whose corresponding altitude has length $h$ is given by $A = \frac{1}{2}bh$.

7.1.4 The area of a right triangle with legs of lengths $a$ and $b$ is given by $A = \frac{1}{2}ab$

7.2.1 (Heron's Formula) If the three sides of a triangle have lengths $a$, $b$, and $c$, then the area $A$ of the triangle is given by $A = \sqrt{s(s-a)(s-b)(s-c)}$ where the semiperimeter of the triangle is $s = \frac{1}{2}(a+b+c)$.

7.2.2 (Brahmagupta's Formula) For a cyclic quadrilateral with sides of lengths $a$, $b$, $c$, and $d$ is given by $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$ where $s = \frac{1}{2}(a+b+c+d)$



7.2.3 The area $A$ of a trapezoid whose bases has lengths $b_1$ and $b_2$ and whose altitude has length $h$ is given by $A = \frac{1}{2}h(b_1 + b_2)$.

7.2.4 The area $A$ of any quadrilateral with perpendicular diagonals of lengths $d_1$ and $d_2$ is given by $A = \frac{1}{2}d_1 d_2$.

7.2.5 The area $A$ of a rhombus whose diagonals have lengths $d_1$ and $d_2$ is given by $A = \frac{1}{2}d_1 d_2$.

7.2.6 The area $A$ of a kite whose diagonals have lengths $d_1$ and $d_2$ is given by $A = \frac{1}{2}d_1 d_2$.

7.27 The ratio of the areas of two similar triangles equals the square of the ratio of the lengths of any two corresponding sides; that is $\frac{A_1}{A_2} = \left(\frac{a_1}{a_2}\right)^2$.

7.3.1 A circle can be circumscribed about (or inscribed in) any regular polygon.

7.3.2 The measure of the central angle of a regular polygon of $n$ sides is given by $c = \frac{360}{n}$.

7.3.3 Any radius of a regular polygon bisects the angle at the vertex to which it is drawn.

7.3.4 Any apothem to a side of a regular polygon bisects the side of the polygon to which it is drawn.

7.3.5 The area $A$ of a regular polygon whose apothem has length $a$ and whose perimeter is $P$ is given by $A = \frac{1}{2}aP$.

7.4.1 The circumference of a circle is given by the formula $C = \pi d$ or $C = 2\pi r$.

7.4.2 In a circle whose circumference is $C$, the length $\ell$ of an arc whose degree measure is $m$ is given by $\ell = \frac{m}{360} \cdot C$. (For arc $AB$, $l\widehat{AB} = \frac{m\widehat{AB}}{360} \cdot C$)

7.4.3 The area $A$ of a circle whose radius is of length $r$ is given by $A = \pi r^2$.

7.5.1 In a circle of radius $r$, the area $A$ of a sector whose arc has degree measure $m$ is given by $A = \frac{m}{360}\pi r^2$.

7.5.2 The area of a semicircular region of radius $r$ is $A = \frac{1}{2}\pi r^2$.

7.5.3 Where $P$ represents the perimeter of a triangle and $r$ represents the length of the radius of its inscribed circle, the area of the triangle is given by $A = \frac{1}{2}rP$.