

Chapter 6 Notes: Circles

IMPORTANT TERMS AND DEFINITIONS

A **circle** is the set of all points in a plane that are at a fixed distance from a given point known as the center of the circle. Any line segment connecting the center and a point on the circle is called a **radius** of a circle. All radii of a circle are congruent. A **diameter** of a circle is a chord that contains the center of the circle. The length of the diameter is twice that of the radius.

Congruent circles are two or more circles that have congruent radii.

Concentric circles are coplanar circles that have a common center.

A **central angle** of a circle is an angle whose vertex is the center of the circle and whose sides are radii of the circle.

The **intercepted arc** of an angle is determined by the two points of intersection of the angle with the circle and all points of the arc in the interior of the angle.

In a circle or congruent circles, **congruent arcs** are arcs with equal measures.

The sum of the measures of the consecutive arcs that form a circle is exactly 360° .

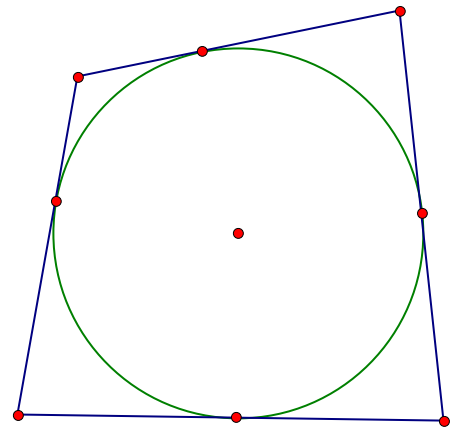
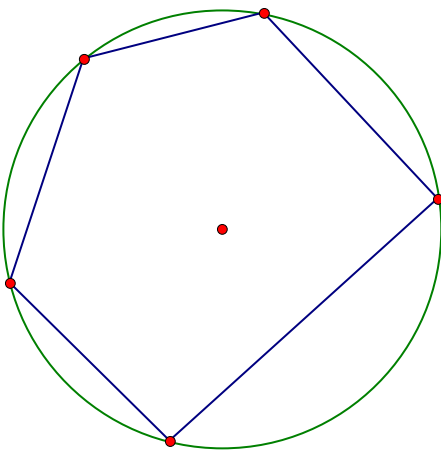
If $m\widehat{AB} = m\widehat{BC}$, then point B is the **midpoint** of \widehat{ABC} and \widehat{ABC} is **bisected** at point B .

An **inscribed angle** of a circle is an angle whose vertex is a point on the circle and whose sides are chords of the circle.

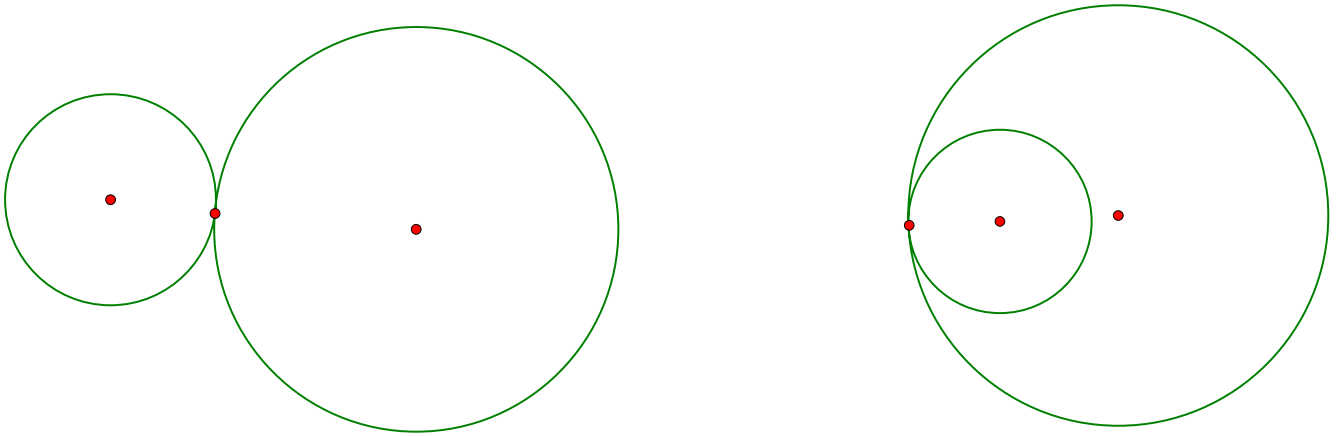
A **tangent** is a line that intersects a circle at exactly one point; the point of intersection is the **point of contact** or the **point of tangency**.

A **secant** is a line (or segment or ray) that intersects the circle at exactly two points.

A polygon is **inscribed in a circle** if its vertices are points on the circle and its sides are chords of the circle. Equivalently, the circle is said to be **circumscribed about the polygon**.

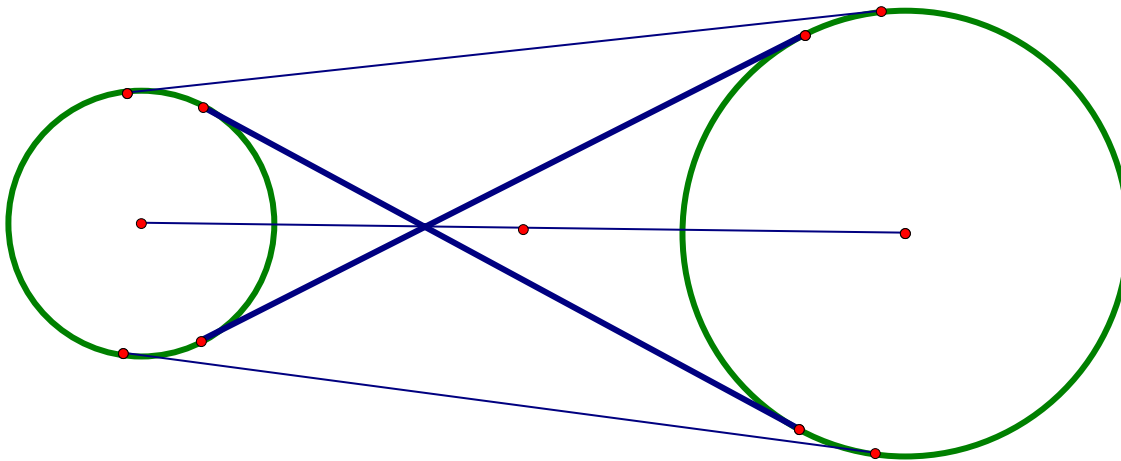


A polygon is **circumscribed about a circle** if all sides of the polygon are segments tangent to the circle; also the circle is said to be **inscribed in the polygon**.



If two circles touch at one point, they are **tangent circles**. Two circles are **internally tangent** if one of the tangent circles lie inside the other. Two circles are **externally tangent** if each of the tangent circles lie outside the other.

The line (or line segment) that passes through the centers of two distinct circles is called the **line of centers** of two circles.



A line segment that is tangent to each of two circles is a **common tangent** for these circles. If the common tangent *does not* intersect the line of centers, it is a **common external tangent**. If the common tangent *does* intersect the line of centers for two circles, it is a **common internal tangent** for the two circles.

A **locus** is the set of all points and only those points that satisfy a given condition (or set of conditions). A number of lines are **concurrent** if they have exactly one point in common.

If a circle is inscribed in a triangle, the sides of the triangle are tangents for the inscribed circle and the circle is called the **incircle** of the triangle.

If a circle is circumscribed about a triangle, the sides of the triangle are chords of the circle and the circle is called the **circumcircle** of the triangle.

POSTULATES:

16. (**Central Angle Postulate**) In a circle, the degree measure of a central angle is equal to the degree measure of its intercepted arc.
17. (**Arc Addition Postulate**) If B lies between A and C on a circle, then $m\widehat{AB} + m\widehat{BC} = m\widehat{ABC}$.

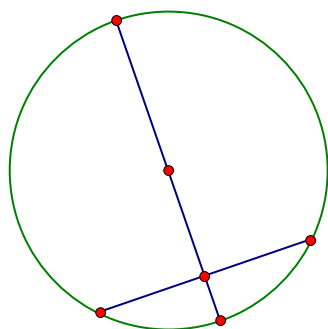
CONSTRUCTIONS:

8. To construct a tangent to a circle at a point on the circle.
9. To construct a tangent to a circle from an external point.

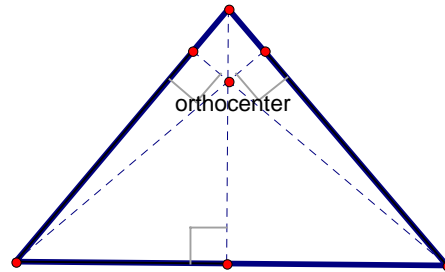
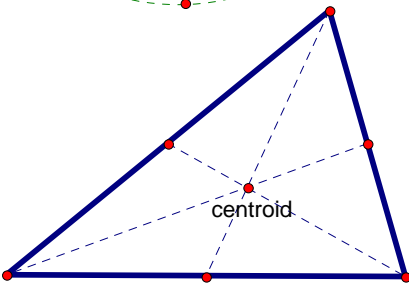
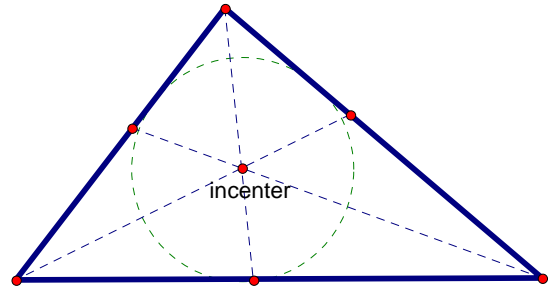
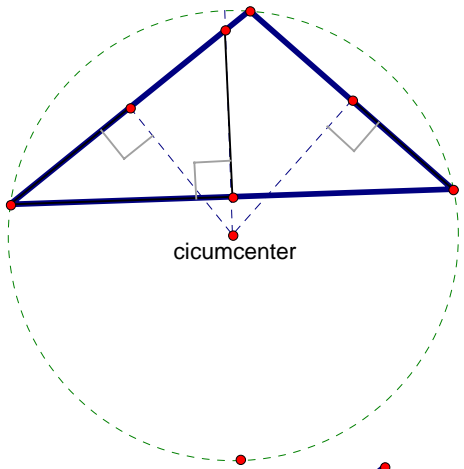
THEOREMS AND COROLLARIES:

- 6.1.1 A radius that is perpendicular to a chord bisects the chord.
- 6.1.2 The measure of an inscribed angle of a circle is one-half the measure of its intercepted arc.
- 6.1.3 In a circle or in congruent circles, congruent minor arcs have congruent central angles.
- 6.1.4 In a circle or in congruent circles, congruent central angles have congruent arcs.
- 6.1.5 In a circle or in congruent circles, congruent chords have congruent minor (major) arcs.
- 6.1.6 In a circle or in congruent circles, congruent arcs have congruent chords.
- 6.1.7 Chords that are at the same distance from the center of a circle are congruent.
- 6.1.8 Congruent chords are located at the same distance from the center of a circle.
- 6.1.9 An angle inscribed in a semicircle is a right angle.
- 6.1.10 If two inscribed angles intercept the same arc, then these angles are congruent.
- 6.2.1 If a quadrilateral is inscribed in a circle, the opposite angles are supplementary.
- 6.2.2 The measure of an angle formed by two chords intersecting within a circle is one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
- 6.2.3 The radius (or any other line through the center of a circle) drawn to a tangent at the point of tangency is perpendicular to the tangent at that point.
- 6.2.4 The measure of an angle formed by a tangent and a chord drawn to the point of tangency is one-half the measure of the intercepted arc.
- 6.2.5 The measure of an angle formed when two secants intersect at a point outside the circle is one-half the difference of the measures of the two intercepted arcs.
- 6.2.6 If an angle is formed by a secant and a tangent that intersect in the exterior of the circle, then the measure of the angle is one-half the difference of the measures of the intercepted arcs.
- 6.2.7 If an angle is formed by two intersecting tangents, then the measure of the angle is one-half the difference of the measures of the intercepted arcs.
- 6.2.8 If two parallel lines intersect a circle, the intercepted arcs between these lines are congruent.
- 6.3.1 If a line is drawn through the center of a circle perpendicular to a chord, then it bisects the chord and its arc.

- 6.3.2 If a line through the center of a circle bisects a chord other than the diameter, then it is perpendicular to the chord.



- 6.3.3 The perpendicular bisector of a chord contains the center of the circle.
- 6.3.4 The tangent segments to a circle from an external point are congruent.
- 6.3.5 If two chords intersect within a circle, then the product of the lengths of the segments (parts) of one chord is equal to the product of the lengths of the segments of the other chord.
- 6.3.6 If two secant segments are drawn to a circle from an external point, then the products of the lengths of each secant with its external segment are equal.
- 6.3.7 If a tangent segment and a secant segment are drawn to a circle from an external point, then the square of the length of the tangent equals the product of the length of the secant with the length of its external segment.
- 6.4.1 The line that is perpendicular to the radius of a circle at its endpoint on the circle is tangent to the circle.
- 6.4.2 In a circle (or in congruent circles) containing two unequal central angles, the larger angle corresponds to the larger intercepted arc.
- 6.4.3 In a circle (or in congruent circles) containing two unequal arcs, the larger arc corresponds to the larger central angle.
- 6.4.4 In a circle (or in congruent circles) containing two unequal chords, the shorter chord is at the greater distance from the center of the circle.
- 6.4.5 In a circle (or in congruent circles) containing two unequal chords, the chord nearer the center of the circle has the greater length.
- 6.4.6 In a circle (or in congruent circles) containing two unequal chords, the longer chord corresponds to the greater minor arc.
- 6.4.7 In a circle (or in congruent circles) containing two unequal minor arcs, the greater minor arc corresponds to the longer of the chords related to these arcs.
- 6.5.1 The locus of points in a plane equidistant from the sides of an angle is the angle bisector.
- 6.5.2 The locus of points in a plane that are equidistant from the endpoints of a line segment is the perpendicular bisector of that line segment.



6.6.1 The three angle bisectors of the angles of a triangle are concurrent. (The point of concurrency is called the **incenter**.)

6.6.2 The three perpendicular bisectors of the sides of a triangle are concurrent. (The point of concurrency is called the **circumcenter**.)

6.6.2 The three altitudes of a triangle are concurrent. (The point of concurrency is called the **orthocenter**.)

6.6.4 The three medians of a triangle are concurrent at a point that is two-thirds the distance from any vertex to the midpoint of the opposite side. (The point of concurrency is called the **centroid**.)