

Chapter 1 Notes: Line and Angle Relationships

IMPORTANT TERMS AND DEFINITIONS

A **statement** is a group of words and symbols that can be classified collectively as true or false. Here are some examples:

1. Conjunction: Jessica is six years old and weighs 56 pounds. $[P \wedge Q]$ (True only if both P and Q are true)
2. Disjunction: Alan will visit Walt Disney World or he will spend a week in the Bahamas. $[P \vee Q]$ (True if at least one of P or Q is true)
3. Negation: "Some birds cannot fly." is the negation of "All birds can fly." $[\sim P]$ (True if P is false.)
4. Conditional or implication: If Sam visits New Orleans, then he has been to Louisiana. $[P \rightarrow Q]$ (False only if the promise "Q" is not kept.)
5. Converse: If Sam has been to Louisiana, then he has been to New Orleans. $[Q \rightarrow P]$
6. Inverse: If Sam does not visit New Orleans, then he has not been to Louisiana. $[\sim P \rightarrow \sim Q]$
7. Contrapositive: If Sam has not been to Louisiana, then he has not been to New Orleans. $[\sim Q \rightarrow \sim P]$

Types of thinking or reasoning used to develop mathematical principles:

1. **Intuition**: An inspiration leading to the statement of a theory. (With intuition, a sudden insight allows one to make a statement without applying any formal reasoning.)
2. **Induction**: An organized effort to test the theory. (Use specific observations and experiments to draw a general conclusion.)
3. **Deduction**: A formal argument that proves the tested theory. **Deduction** is the type of reasoning in which the knowledge and acceptance of selected assumptions guarantees the truth of a particular conclusion.

Law of Detachment: Let P and Q represent simple statements, and assume that statements 1 and 2 are true. Then a valid argument having conclusion C has the form

1. If P, then Q.
2. P

- C. \therefore Q

1. If a man lives in San Francisco, then he lives in California.
2. Ken lives in San Francisco.

- C. Therefore, Ken lives in California.

Law of Negative Inference:

1. $P \rightarrow Q$ Premise 1
2. $\sim Q$ Premise 2

- C. $\sim P$ Conclusion

1. If a man lives in Houston, then he lives in Texas.
2. Tim does not live in Texas.

- C. Tim does not live in Houston.

Law of Syllogism:

1. $P \rightarrow Q$ Premise 1
2. $Q \rightarrow R$ Premise 2

- C. $P \rightarrow R$ Conclusion

Four Parts of a Mathematical System:

1. Undefined Terms
2. Defined Terms
3. Axioms and Postulates
4. Theorems

Undefined Terms: point, line, plane

Formal Definitions:

1. An *isosceles triangle* is a triangle that has two congruent sides.
2. A *line segment* is the part of a line that consists of two points, known as the endpoints, and all the points between them.
3. The *distance* between two points A and B is the length of the line segment that joins the two points.
4. *Congruent* (\cong) *segments* are two segments that have the same length.
5. The midpoint of a line segment is the point that separates the line segment into two congruent parts.
6. *Ray* AB, denoted by \overrightarrow{AB} , is the union of \overline{AB} and all the points X on \overleftrightarrow{AB} such that B is between A and X.
7. *Parallel lines* are lines that lie in the same plane but do not intersect.
8. An *angle* is the union of two rays that share a common endpoint. An angle whose measure is less than 90° is an *acute angle*. If the angle's measure is exactly 90° , the angle is a *right angle*. If the angle's measure is between 90° and 180° , the angle is *obtuse*. Finally, an angle whose measure is exactly 180° is a *straight angle*.
9. Two angles are *adjacent* if they share a common side and a common vertex but have no interior points in common.
10. *Congruent angles* ($\cong \sphericalangle s$) are two angles with the same measure.
11. The bisector of an angle is the ray that separates the given angle into two congruent angles.
12. Two angles are *complementary* if the sum of their measures is 90° . Each angle is the *complement* of the other. Similarly, two angles are *supplementary* if the sum of their measures is 180° . Each angle is the *supplement* of the other.
13. When two straight lines intersect, the pairs of nonadjacent angles formed are each known as *vertical angles*.
14. *Perpendicular lines* are two lines that meet to form congruent adjacent angles.

ALGEBRAIC PROPERTIES:

Addition Property of Equality: If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality: If $a = b$, then $a - c = b - c$.

Multiplication Property of Equality: If $a = b$, then $ac = bc$.

Division Property of Equality: If $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.

Distributive Property: $a(b + c) = ab + ac$

Substitution Property: If $a = b$, then a replaces b in any equation.

Transitive Property: If $a = b$ and $b = c$, then $a = c$.

POSTULATES:

1. Through two distinct points, there is exactly one line.
2. (Ruler Postulate) The measure of any line segment is a unique positive number.
3. (Segment-Addition Postulate) If X is a point on \overline{AB} and A - X - B (i.e. X is between A and B), then $AX + XB = AB$.
4. If two lines intersect, they intersect at a point.
5. Through three noncollinear points, there is exactly one plane.
6. If two distinct planes intersect, then their intersection is a line.
7. Given two distinct points in a plane, the line containing these points also lies in the plane.
8. (Protractor Postulate) The measure of an angle is a unique positive number.
9. (Angle-Addition Postulate) If a point D lies in the interior of angle ABC , then $m\angle ABD + m\angle DBC = m\angle ABC$.

CONSTRUCTIONS:

1. To construct a line segment congruent to a given line segment.
2. To construct the midpoint M of a given line segment AB .
3. To construct an angle congruent to a given angle.
4. To construct the angle bisector of a given angle.
5. To construct the line perpendicular to a given line at a specified point on the given line.

THEOREMS AND COROLLARIES:

- 1.3.1 The midpoint of a line segment is unique.
- 1.4.1 There is one and only one angle bisector for a given angle.
- 1.6.1 If two lines are perpendicular, then they meet to form right angles.
- 1.6.2 If two lines intersect, then the vertical angles formed are congruent.
- 1.6.3 There is exactly one line perpendicular to a given line at any point on the line.
- 1.6.4 The perpendicular bisector of a line segment is unique
- 1.7.1 If two lines meet to form a right angle, then these lines are perpendicular.
- 1.7.2 If two angles are complementary to the same angle (or to congruent angles), then these angles are congruent.
- 1.7.3 If two angles are supplementary to the same angle (or to congruent angles), then these angles are congruent.
- 1.7.4 Any two right angles are congruent.
- 1.7.5 If the exterior sides of two adjacent acute angles form perpendicular rays, then these angles are complementary.
- 1.7.6 If the exterior sides of two adjacent angles form a straight line, then these angles are supplementary. (The two angles form a linear pair.)
- 1.7.7 If two segments are congruent, then their midpoints separate these segments into four congruent segments.
- 1.7.8 If two angles are congruent, then their bisectors separate these angles into four congruent angles.