Math 275 **Exam 2 Review** **Answer Key**

1. Find Wronskian of f and xf : Since $W(x) = f^2 \neq 0$ on [a, b], then f and xf are linearly independent.

2. Suppose the set of functions is linearly dependent. Then one of the functions is a linear combination of the others, that is $e^{4x} = Axe^{4x} + Bx^2e^{4x} + Cx^3e^{4x}$ for some constants $A,\ B,$ and C. Since $e^{4x} \neq 0$, divide both sides by e^{4x} obtaining $1 = Ax + Bx^2 + Cx^3$. This means that the functions $1,\ x,\ x^2,\ x^3$ are linearly dependent, but they are linearly independent since its Wronskian (find it) $W(x) = 12 \neq 0$. So the original set of functions must be linearly independent.

3. a. linearly independent since its Wronskian is t^3 which is not zero on (0, 1)
   b. linearly dependent since $\csc^2 x = 1 + \cot^2 x$

4. a. $xD^3 + (2+3x)D^2 + 6D$
   b. $x^3D^2 + (6x^2 - 3x)D - 12$

5. a. $y = c_1e^{-2x} + c_2e^{\frac{5}{2}x} + c_3e^{-\frac{1}{2}x}$
   b. $y = c_1e^{-x} + (c_2 + c_3x + c_4x^2)e^{\frac{1}{3}x}$
   c. $y = c_1 + c_2x + c_3x^2 + e^{-3x}(c_4 \cos x + c_5 \sin x)$
   d. $y = (c_1 + c_2x)\cos 2x + (c_3 + c_4x)\sin 2x$
   e. $y = c_1e^{2x} + e^{-x}(c_2 \cos\sqrt{3}x + c_3 \sin\sqrt{3}x)$
   f. $y = c_1 + c_2x + c_3 \cos 2x + c_4 \sin 2x + c_5 \cosh x + c_6 \sinh x$
   g. $y = (c_1 + c_2x)e^x + c_3 \cos x + c_4 \sin x$
   h. $y = (c_1 + c_2x + c_3x^2)e^{-x} + (c_4 + c_5x)e^{\frac{3}{2}x}$

6. a. $y = e^{-2x}(c_1 \cos x + c_2 \sin x) - 8 + 10x + \frac{1}{2}e^{3x}$
   b. $y = c_1 \cos x + c_2 \sin x + c_3e^x + x\cos - x\sin x$
   c. $y = c_1e^{2x} + c_2e^{-x} + \frac{3}{2} - 3x - 2xe^{-x}$
   d. $y = c_1e^x + c_2e^{-x} + c_3 \cos x + c_4 \sin x - 7x^2$
   e. $y = c_1 + c_2x + e^x(c_3 \cos 2x + c_4 \sin 2x) + e^{-x}(-\frac{18}{65}\cos x + \frac{1}{65}\sin x)$
   f. $y = e^{-3x}(c_1 \cos 2x + c_2 \sin 2x) + 2 + 4\cos x + 2\sin x$

7. a. $y = c_1 \cos 4x + c_2 \sin 4x - \frac{10}{9}\cos 5x - \frac{5}{2}$
   b. $y = c_1 \cos x + c_2 \sin x + c_3e^{\sqrt{6}x} + c_4e^{-\sqrt{6}x} - \frac{2}{5}e^{3x} - \frac{4}{3} + 2x$
   c. $y = c_1 + c_2x + c_3x^2 + c_4 \cos 3\sqrt{2}x + c_5 \sin 3\sqrt{2}x - \frac{5}{18}x^3 - \frac{3}{88}e^{-2x}$
   d. $y = c_1 \cos 2x + c_2 \sin 2x + \frac{4}{3}\cos x + \frac{12}{5}\sin 3x$
   e. $y = c_1 + c_2e^{3x} + c_3e^{-3x} + \frac{3}{14}e^{4x} + \frac{8}{3}x$

8. a. $y = -20e^{-\frac{1}{2}x} + 21 - 10x + 3x^2$
   b. $y = -2\cos x - 4\sin x + 2e^{2x}$
   c. $y = e^{-x} - \cos 2x$
   d. $y = -6 + 6x + 3e^{-2x}$
   e. $y = -2 + 2\cosh x$ or $y = -2 + e^x + e^{-x}$