

Solve each equation by using an appropriate method.

1.  $\frac{dy}{dx} = \sec^2 x \sec^3 y$ ; separable

Solution:  $3 \sin y - \sin^3 y = 3 \tan x + C$

2.  $(x + y)dx + x dy = 0$ ; homogeneous, exact, 1<sup>st</sup> order linear, by inspection

Solution:  $x^2 + 2xy = C$

3.  $(\cos x \cos y - \cot x)dx - \sin x \sin y dy = 0$ ; Exact

Solution:  $\sin x \cos y - \ln |\sin x| = C$

4.  $2(2xy + 4y - 3)dx + (x + 2)^2 dy = 0$ ; Determine IF, 1<sup>st</sup> order linear

Solution:  $y(x + 2)^4 - 2(x + 2)^3 = C$

5.  $y(2 - 3xy)dx - x dy = 0$ ; Bernoulli

Solution:  $y^{-1}x^2 = x^3 + C$

6.  $2y(x + y + 2)dx + (y^2 - x^2 - 4x - 1)dy = 0$ ; Determine IF

Solution:  $x^2 + 2xy + 4x + y^2 + 1 = Cy$

7.  $4(3x + y - 2)dx - (3x + y)dy = 0$ ,  $x = 1, y = 0$ ; Substitution

Solution:  $28(x - 1) - 7y - 8 \ln(|21x + 7y - 8| - \ln 13) = 0$

8.  $(2x + 3y - 5)dx + (3x - y - 2)dy = 0$ ; linear coefficients, exact

Solution:  $2x^2 + 6xy - 10x - y^2 - 4y = C$

9.  $xy dx + (y^4 - 3x^2)dy = 0$ ; Determine IF

Solution:  $x^2 - y^4 = Cy^6$

10.  $\left[ x \exp\left(\frac{y^2}{x^2}\right) - y \right] dx + x dy = 0$ ,  $x = 1, y = 2$  (Hint: Use power series.);

by inspection, homogeneous: Solution:  $x = \exp\left[\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)n!} \left(2^{n+1} - \frac{y^{2n+1}}{x^{2n+1}}\right)\right]$

11.  $(x + y - 4)dx - (3x - y - 4)dy = 0$ ,  $x = 4, y = 1$ ; linear coefficients

Solution:  $(x - y)\ln|x - y| = (-3\ln 3)y + 6\ln 2$

12.  $(2y^3 - x^3)dx + 3xy^2dy = 0$ ,  $x = 1, y = 1$ ; homogeneous, Determine IF

Solution:  $5x^2y^3 - x^5 = C$

13.  $2xyy' = y^2 - 2x^3$ ,  $x = 1, y = 2$ ; Determine IF

Solution:  $y^2 + x^3 = 5x$

14.  $y(2x - y + 1)dx + x(3x - 4y + 3)dy = 0$ ; Determine IF

Solution:  $x^2y^3 - xy^4 + xy^3 = C$

15.  $y' \tan x \sin 2y = \sin^2 x + \cos^2 y$ ; Substitution

Solution:  $3\sin^2 x \cos^2 y = -6\sin x + 2\sin^3 x + C$

16.  $x^4y' = -x^3y - \csc(xy)$ ; by inspection

Solution:  $2x^2 \cos(xy) + 1 = Cx^2$

17.  $y \ln x \ln y dx + dy = 0$ ; separable

Solution:  $x \ln|x| - x + \ln|\ln|y|| = C$

18.  $y(x^2 + y^2)dx + x(3x^2 - 5y^2)dy$ ,  $x = 2, y = 1$ ; homogeneous

Solution:  $xy^3(x + y) = 6(x - y)$

19.  $(xy^2 + x - 2y + 3)dx + x^2ydy = 2(x + y)dy$ ,  $x = 1, y = 1$ ; Exact

Solution:  $x^2y^2 + x^2 - 4xy + 6x - 2y^2 = C$

20.  $(y - \sin^2 x)dx + \sin x dy = 0$ ; 1<sup>st</sup> order linear

Solution:  $(\csc x - \cot x)y = x - \sin x + C$