Math 270 **Exam 2 Review Answer Key**

1. T F F F T T F F T T

2. a. Not a vector space. Not closed under $\oplus$ b. Yes
   c. Not a vector space. $r \odot \left(s \odot \vec{u}\right) \neq rs \odot \vec{u}$

3. a. Yes b. No c. Yes

4. a. Yes b. No c. Yes

5. Yes

6. a. No b. Yes c. No

7. a. No b. Yes c. Yes

8. $\left\{\begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \end{bmatrix}\right\}$

9. Let $a_1\overrightarrow{w_1} + a_2\overrightarrow{w_2} + a_3\overrightarrow{w_3} = \vec{0}$. Then $a_1\left(\overrightarrow{v_1}\right) + a_2\left(\overrightarrow{v_1} + \overrightarrow{v_3}\right) + a_3\left(\overrightarrow{v_1} + \overrightarrow{v_2} + \overrightarrow{v_3}\right) = \vec{0}$. This means $\left(a_1 + a_2 + a_3\right)\overrightarrow{v_1} + a_3\overrightarrow{v_2} + \left(a_2 + a_3\right)\overrightarrow{v_3} = \vec{0}$. Since $\left\{\overrightarrow{v_1}, \overrightarrow{v_2}, \overrightarrow{v_3}\right\}$ is linearly dependent, the coefficients $a_1 + a_2 + a_3$, a_3 and $a_2 + a_3$ are not all zeros. This implies that a_1, a_2, a_3 are not all zeros. Therefore, $\left\{\overrightarrow{w_1}, \overrightarrow{w_2}, \overrightarrow{w_3}\right\}$ is linearly dependent.

10. a. No b. Yes c. No

11. $\left\{\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}\right\}$ spans W and $\dim W = 2$

12. $\left\{2t^2 + t, \ -3t^2 + 1\right\}$ is a basis for W and $\dim W = 2$

13. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ forms a basis for R^4

14. Let A be a singular matrix. Then the homogeneous system $A\vec{x} = \vec{0}$ has a nontrivial solution $\vec{x} \neq \vec{0}$. Since $\left\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_n}\right\}$ is a linearly independent set of vectors in R^n (which has dimension *n*), it is also a basis for R^n. Thus there exists $a_1, a_2, \ldots, a_n$ (not all zeros) such that $\vec{x} = a_1\vec{v_1} + a_2\vec{v_2} + \ldots + a_n\vec{v_n}$. Now, substituting $\vec{x} = a_1\vec{v_1} + a_2\vec{v_2} + \ldots + a_n\vec{v_n}$ into $A\vec{x} = \vec{0}$, we get $A\left(a_1\vec{v_1} + a_2\vec{v_2} + \ldots + a_n\vec{v_n}\right) = a_1A\vec{v_1} + a_2A\vec{v_2} + \ldots + a_nA\vec{v_n} = \vec{0}$. Since not all a_i's are zero, we conclude that $\left\{A\vec{v_1}, A\vec{v_2}, \ldots, A\vec{v_n}\right\}$ is linearly dependent.

15. $\left\{ \begin{bmatrix} 5 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$ forms a basis for the solution space of $A\vec{x} = \vec{0}$ with dimension 2.

16. $\left[\vec{v}\right]_S = P_{S\leftarrow T}\left[\vec{v}\right]_T = \begin{bmatrix} 4 & 1 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 \\ 13 \end{bmatrix}$

17. S is linearly dependent since $e^t - e^{-t} = 1e^t + (-1)e^{-t}$. Take $T = \left\{t, e^t, e^{-t}\right\}$ and show that it is linearly independent, so it forms a basis for V. That is, let $at + be^t + ce^{-t} = 0$. Take 1st and 2nd derivatives: $a + be^t - c^{-t} = 0$ (this implies $a = 0$) and $be^t + ce^{-t} = 0$. Adding them together, we get $2be^t = 0$ which implies that $b = c = 0$. Thus, T is linearly independent. The dimension of V is 3 since its basis T has 3 vectors. But R_3 is also of dimension 3, so V and R_3 are isomorphic.

18. a. Basis = $\left\{ \begin{bmatrix} 1 \\ -1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 2 \\ -5 \end{bmatrix} \right\}$; rank = 2, nullity = 2

b. Basis= $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \end{bmatrix} \right\}$; rank = 3, nullity = 1