Math 270 **Exam 2 Review**

1. Mark the following as TRUE or FALSE.
   a. If A is a nonsingular matrix, then the null space of A is $\{\vec{0}\}$.
   b. The set of all solutions to the linear system $A\vec{x}=\vec{b}$, $\vec{b}\neq\vec{0}$ is a subspace of R^n.
   c. The set $\left\{\begin{bmatrix}1\\0\\2\\1\end{bmatrix}, \begin{bmatrix}-2\\1\\0\\-3\end{bmatrix}, \begin{bmatrix}0\\1\\2\\1\end{bmatrix}\right\}$ spans R^4.
   d. Let S_1 and S_2 be finite subsets of a vector space such that S_1 is a subset of S_2. If S_1 is linearly independent, so is S_2.
   e. Any set of vectors containing the zero vector is linearly dependent.
   f. If S spans an *n*-dimensional vector space V then S has at least *n* elements.
   g. If S is a linearly independent set of vectors in a finite-dimensional vector space V, then S is basis for V.
   h. $\{-t^2+t,\ t+1,\ t^2-1,\ t^2+t+1\}$ is a basis for P_2.
   i. If the rank of a 4×7 matrix A is 4, then the rows of A are linearly independent.
   j. If A is an $n\times n$ matrix, the homogeneous system $A\vec{x}=\vec{0}$ has a nontrivial solution if and only if the columns of A are linearly dependent.

2. Is $(V,\oplus,\odot)$ a vector space? If not, identify a property that is not satisfied.
   a. V is the set of all 4th degree polynomials , $\oplus$ is addition of polynomials, $\odot$ is multiplication by a constant
   b. V is the set of all solutions to the differential equation $y''-3y'-4y=0$, $\oplus$ is addition of functions, $\odot$ is multiplication by a constant
   c. V is the set of all real numbers, $u\oplus v=uv,\ c\odot u=c+u$

3. Which of the following subsets of the vector space M_{nn} are subspaces?
   a. The set of all $n\times n$ skew symmetric matrices.
   b. The set of all $n\times n$ nonsingular matrices.
   c. The set of all $n\times n$ lower triangular matrices.

4. Which of the given subsets of R^3 are subspaces?

a. $\begin{bmatrix} a+c \\ 2a-c \\ c \end{bmatrix}$ b. $\begin{bmatrix} a \\ a+1 \\ a-1 \end{bmatrix}$ c. $\begin{bmatrix} a \\ 0 \\ c \end{bmatrix}$

5. Let W be the set of all 2×2 matrices $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $a+d=b+c$. Is W a subspace of M_{22}?

6. Which of the following sets of vectors span R_4?

a. $\{[1\ \ 2\ \ 1\ \ 0],\ [1\ \ -1\ \ 0\ \ 1],\ [-3\ \ 0\ \ 1\ \ 2]\}$

b. $\{[1\ \ 1\ \ 0\ \ 0],\ [1\ \ 2\ \ -1\ \ 1],\ [0\ \ 0\ \ 1\ \ -1],\ [2\ \ 1\ \ 2\ \ -1]\}$

c. $\{[3\ \ 2\ \ -1\ \ 2],\ [4\ \ 0\ \ 0\ \ 2],\ [1\ \ -2\ \ 1\ \ 0],\ [7\ \ 2\ \ \text{-}1\ \ 4]\}$

7. Which of the given vectors in P_2 are linearly dependent?

a. $t^2-t,\ 3t^2+4t+2$

b. $t^2+t,\ 2t+3,\ t^2-t+1,\ 3t^2+6t+6$

c. $2t^2+t+1,\ 3t^2+t-5,\ t+13$

8. Find a set of vectors spanning the null space of $A=\begin{bmatrix} 1 & 2 & -1 & 1 \\ -2 & 2 & -4 & 4 \\ 0 & 6 & -1 & -4 \\ -1 & 4 & 0 & -5 \end{bmatrix}$.

9. Suppose that $S=\{\vec{v_1},\ \vec{v_2},\ \vec{v_3}\}$ is a linearly dependent set of vectors in a vector space V. Is $T=\{\vec{w_1},\ \vec{w_2},\ \vec{w_3}\}$ where $\vec{w_1}=\vec{v_1},\ \vec{w_2}=\vec{v_1}+\vec{v_3},\ \vec{w_3}=\vec{v_1}+\vec{v_2}+\vec{v_3}$ linearly dependent or linearly independent? Justify your answer.

10. Which of the given subsets form a basis for M_{22}?

a. $\left\{\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}\right\}$

b. $\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}\right\}$

c. $\left\{\begin{bmatrix} -2 & 4 \\ 6 & 4 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} -3 & 2 \\ 5 & 6 \end{bmatrix}, \begin{bmatrix} -2 & -1 \\ 0 & 4 \end{bmatrix}\right\}$

11. Find a basis for the subspace W of R^3 spanned by $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ -1 \end{bmatrix} \right\}$.

What is the dimension of W?

12. Find a basis for the subspace W of P_2 consisting of all vectors of the form $at^2 + bt + c$ where $a - 2b + 3c = 0$. What is the dimension of W?

13. Find a basis for R^4 that includes the vectors $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$.

14. Suppose that $\{\vec{v_1}, \vec{v_2}, \ . \ . \ . \ \vec{v_n}\}$ is a linearly independent set of vectors in R^n and let A be a singular matrix. Prove or disprove that $\{A\vec{v_1}, \mathrm{A}\vec{v_2}, \ . \ . \ . \ \mathrm{A}\vec{v_n}\}$ is linearly independent.

15. Find a basis for and dimension of the solution space of the given homogeneous system:

$$\begin{aligned} x + 2y + 3z - w &= 0 \\ 2x + 3y + 2z \quad &= 0 \\ 3x + 4y + z + w &= 0 \\ x + \ y - z + w &= 0 \end{aligned}$$

16. Let $S = \{2t - 1, \ -t + 1\}$ and $T = \{3t + 1, \ t\}$ be ordered bases for P_1. If $\vec{v}$ is in P_1 and $\left[\vec{v}\right]_T = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ determine $\left[\vec{v}\right]_S$.

17. Let V be the subspace of the vector space of all real-valued functions that is spanned by the set $S = \{t, \ e^t, \ e^{-t}, \ e^t - e^{-t}\}$. Show that V and R_3 are isomorphic.

18. Find a basis for the column space and compute the rank and nullity of each given matrix.

a. $\begin{bmatrix} 1 & 3 & -2 & 4 \\ -1 & 4 & -5 & 10 \\ 3 & 2 & 1 & -2 \\ 3 & -5 & 8 & -16 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$