Math 270 **Exam 1 Review Answer Key**

1. T F T T F T F F T F

2. a. $\begin{bmatrix} -4 & -16 \\ 5 & 16 \\ 3 & 37 \\ -10 & 13 \\ 9 & 39 \end{bmatrix}$ b. $\begin{bmatrix} -18 & 15 & 17 \\ 8 & 10 & 26 \end{bmatrix}$ c. $\begin{bmatrix} 14 & -6 \\ -6 & 35 \end{bmatrix}$

3. a. projection onto the *yz*-plane b. projection onto the *x*-axis

4. a. $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

6. a. Solution: (0, 0 , 0) b. Solution: $\left\{\left(-2+k,-1,8-2k,k\right):\ k \text{ a real number}\right\}$

7. a. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ b. $\begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ -\frac{3}{2} & \frac{5}{2} & -\frac{1}{2} \end{bmatrix}$ c. Inverse does not exist

8. a. $\det(A) = -30$, upper triangular b. $\det(A) = 0$, $r_1 = r_3$
   c. $\det(A) = -7$, reduce to upper triangular

9. $\det(A) = -5 \cdot \det(A) = -15$, row 1 multiplied by -5, row 3 replaced by adding to it -3 times row 2

10. a. $adj(A) = \begin{bmatrix} 1 & 4 & 16 \\ 11 & 2 & 8 \\ 2 & 8 & -10 \end{bmatrix}$ b. $A\left(adj(A)\right) = \begin{bmatrix} 42 & 0 & 0 \\ 0 & 42 & 0 \\ 0 & 0 & 42 \end{bmatrix}$
    c. $\det(A) = 42$ since $A\left(adj\left(A\right)\right) = \det(A)I_3 = 42I_3$

11. <u>Proof</u>: Suppose $AB = AC$, and A is nonsingular. Then A^{-1} exists. Multiplying both sides by A^{-1}, we obtain

$$A^{-1}(AB) = A^{-1}(AC)$$
$$(A^{-1}A)B = (A^{-1}A)C$$
$$I_nB = I_nC$$
$$B = C$$

12. Let $S = \frac{1}{2}(A + A^T) = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 4 & -1 \\ 2 & -1 & -4 \end{bmatrix}$ and $K = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -4 & -1 \\ 4 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$.

Note that $A = S + K$.

13. <u>Proof</u>: Let $f : R^n \to R^m$ be a matrix transformation defined by $f(\vec{u}) = A\vec{u}$ where A is an $m \times n$ matrix. Then for any $\vec{u}$ and $\vec{v}$ in R^n and any real numbers c and d,
$f(c\vec{u} + d\vec{v}) = A(c\vec{u} + d\vec{v}) = A(c\vec{u}) + A(d\vec{v}) = cA\vec{u} + dA\vec{v} = cf(\vec{u}) + df(\vec{v})$.

14. <u>Proof</u>: If $B = PAP^{-1}$, then

$B^2 = (PAP^{-1})^2 = (PAP^{-1})(PAP^{-1}) = (PA(P^{-1}P)AP^{-1}) = PAI_nAP^{-1} = PA^2P^{-1}$,

$B^3 = B^2(PAP^{-1}) = (PA^2P^{-1})(PAP^{-1}) = (PA^2(P^{-1}P)AP^{-1}) = PA^2I_nAP^{-1} = PA^3P^{-1}$.

Now suppose that for any positive integer k, $B^k = PA^kP^{-1}$, then ,

$B^{k+1} = B^k(PAP^{-1}) = (PA^kP^{-1})(PAP^{-1}) = (PA^k(P^{-1}P)AP^{-1}) = PA^kI_nAP^{-1} = PA^{k+1}P^{-1}$

By induction $B^k = PA^kP^{-1}$ for any positive integer k.

15. <u>Proof</u>: Let matrix A be skew symmetric and nonsingular. Then A^{-1} exists and $A^T = -A$. Thus $(A^{-1})^T = (A^T)^{-1} = (-A)^{-1} = -A^{-1}$, proving that A^{-1} is also skew symmetric.

16. <u>Proof</u>: Suppose A is idempotent, that is $A^2 = A$. Then
$(A^T)^2 = A^TA^T = (AA)^T = (A^2)^T = A^T$, proving that A^T is also idempotent.

17. <u>Proof</u>: Suppose that $\vec{u}$ and $\vec{v}$ are solutions to the linear system $A\vec{x} = \vec{b}$. This means that $A\vec{u} = \vec{b}$ and $A\vec{v} = \vec{b}$. So $A(\vec{u} - \vec{v}) = A\vec{u} - A\vec{v} = \vec{b} - \vec{b} = 0$. Therefore $\vec{u} - \vec{v}$ is a solution to the associated homogeneous system $A\vec{x} = O$.

18. $p(x) = \frac{9}{2}x^2 - 3x + 1$