Math 270 **Exam 1 Review**

1. Mark the following as TRUE or FALSE.
   a. Homogeneous systems are always consistent.
   b. If A and B are $n \times n$ matrices with no zero entries, then $AB \neq O$.
   c. The sum of two symmetric matrices is symmetric.
   d. $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ defines a matrix transformation that reflects the vector $\begin{bmatrix} x \\ y \end{bmatrix}$ about the x-axis.
   e. Every matrix in row echelon form is also in reduced row echelon form.
   f. Any matrix equivalent to an identity matrix is nonsingular.
   g. If B is the reduced row echelon form of A, then $\det(B) = \det(A)$.
   h. If $\det(A) = 0$, then A has at least two equal rows.
   i. $\det\left(AB^T A^{-1}\right) = \det\left(B\right)$
   j. The determinant of an elementary matrix is always equal to 1.

2. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 2 \\ -2 & 1 & 4 \\ 3 & 2 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ $\quad B = \begin{bmatrix} 2 & -1 \\ -3 & 3 \\ 1 & 5 \end{bmatrix}$ $\quad C = \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ 2 & 4 \\ -3 & 5 \\ 4 & 6 \end{bmatrix}$

   Find: a. $AB + 3C$ b. $C^T A - 3B^T$ c. $B^T B$

3. Give a geometric description of the matrix transformation $f : R^3 \to R^2$ defined by $f\left(\vec{u}\right) = A\vec{u}$.

   a. $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4. Find the reduced row echelon form of the given matrices.

   a. $A = \begin{bmatrix} 1 & 1 & 5 & 3 \\ 2 & -2 & -2 & -1 \\ -3 & 1 & -3 & 2 \end{bmatrix}$ b. $B = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 4 & -1 \\ 5 & 6 & -3 \\ -2 & -2 & 2 \end{bmatrix}$

5. Find a matrix of the form $\begin{bmatrix} I_r & O_{r\,n-r} \\ O_{m-r\,r} & O_{m-r\,n-r} \end{bmatrix}$ that is equivalent to $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 1 & 0 & 2 & 3 \\ 3 & 4 & 8 & 1 \end{bmatrix}$.

6. Solve the given linear systems by Gauss-Jordan reduction

a.
$$\begin{aligned} 3x+y+3z&=0\\ -2x+2y-4z&=0\\ 2x-3y+5z&=0 \end{aligned}$$

b.
$$\begin{aligned} x+y+z&=1\\ x+y-2z&=3\\ 2x+y+z&=2 \end{aligned}$$

c.
$$\begin{aligned} x+y+2z+3w&=13\\ x-2y+z+w&=8\\ 3x+y+z-w&=1 \end{aligned}$$

7. Find the inverse of the given matrix, if it exists.

a. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

b. $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 2 & -3 & 1 \\ -1 & 3 & -3 & -2 \\ 2 & 0 & 1 & 5 \\ 3 & 1 & -2 & 5 \end{bmatrix}$

8. Compute the following determinants via reduction to triangular form, or by citing a particular theorem or corollary.

a. $\begin{vmatrix} 3 & 0 & 0 \\ -8 & 5 & 0 \\ 8 & -3 & -2 \end{vmatrix}$ b. $\begin{vmatrix} -1 & 2 & 3 & -4 \\ 5 & 0 & 4 & 3 \\ -1 & 2 & 3 & -4 \\ 3 & 4 & 0 & 5 \end{vmatrix}$ c. $\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 0 & 1 & 0 \end{vmatrix}$

9. If $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 3$ find $\begin{vmatrix} -5a_1 & -5a_2 & -5a_3 \\ b_1 & b_2 & b_3 \\ -3b_1+c_1 & -3b_2+c_2 & -3b_3+c_3 \end{vmatrix}$.

10. Let $A = \begin{bmatrix} -2 & 4 & 0 \\ 3 & -1 & 4 \\ 2 & 0 & -1 \end{bmatrix}$. Find: a. adj (A) b. $A(\text{adj } A)$ c. $\det(A)$

11. Prove that if $AB = AC$ and A is nonsingular, then $B = C$.

12. Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 5 & 4 & 0 \\ 3 & -2 & -4 \end{bmatrix}$. Find symmetric matrix S and skew symmetric matrix K such that $A = S + K$. (Hint: Recall that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric.)

13. Let $f : R^n \to R^m$ be a matrix transformation defined by $f\left(\vec{u}\right) = A\vec{u}$ where A is an $m \times n$ matrix. Show that $f\left(c\vec{u} + d\vec{v}\right) = cf\left(\vec{u}\right) + df\left(\vec{v}\right)$ for any $\vec{u}$ and $\vec{v}$ in R^n and any real numbers c and d.

14. If $B = PAP^{-1}$, express $B^2,\ B^3,\ B^4,\ \ldots B^k$ where k is a positive integer, in terms of $A,\ P,$ and P^{-1}.

15. If A is skew symmetric and nonsingular, then A^{-1} is also skew symmetric.

16. If A is an $n \times n$ matrix, then A is called <u>idempotent</u>, if $A^2 = A$. Show that if A is idempotent, then A^T is also idempotent.

17. Show that if $\vec{u}$ and $\vec{v}$ are solutions to the linear system $A\vec{x} = \vec{b}$, then $\vec{u} - \vec{v}$ is a solution to the associated homogeneous system $A\vec{x} = O$.

18. Construct a linear system of equations to determine a quadratic polynomial $p(x) = ax^2 + bx + c$ that satisfies the conditions $p(0) = f(0),\ \ p'(0) = f'(0)$, and $p''(0) = f''(0)$, where $f(x) = e^{-3x}$.