Math 270 **Notes on Chapters 1 to 3** **Kolman and Hill, 9th edition**

Definitions

1. The **transpose** of $A=\left[a_{ij}\right]$, is $A^T=\left[a_{ji}\right]$, obtained from *A* by interchanging the rows and columns of *A*.
2. An $n\times n$ matrix $A=\left[a_{ij}\right]$ is called **upper triangular** if $a_{ij}=0$ for $i>j$. It is called **lower triangular** if $a_{ij}=0$ for $i<j$.
3. A matrix *A* with real entries is called **symmetric** if $A^T=A$.
4. A matrix *A* with real entries is called **skew symmetric** if $A^T=-A$.
5. An $n\times n$ matrix *A* is called **nonsingular**, or **invertible**, if there exists an $n\times n$ matrix *B* such that $AB=BA=I_n$; such a *B* is called an inverse of *A*. Otherwise, *A* is called **singular**, or **noninvertible**.
6. The **adjoint** of $n\times n$ matrix $A=\left[a_{ij}\right]$ is the matrix $adj(A)=\left[A_{ji}\right]$ where A_{ji} is the cofactor of a_{ji}.

Important Terminologies

Row (column) echelon form of a matrix
Reduced row (column) echelon form of a matrix
Gaussian elimination
Gauss-Jordan reduction
Augmented matrix
Nonhomogeneous linear system
Homogeneous system
Elementary row (column) operation
Elementary matrix
Equivalent matrices
Determinants
Trace
Reduction to triangular form
Minor of a_{ij}, Cofactor of a_{ij}
Expansion along a row or column
Cramer's Rule

The following statements are equivalent:

1. An $n\times n$ matrix A is nonsingular.
2. The homogeneous system $A\vec{x}=0$ has only the trivial solution
3. *A* is row (column) equivalent to I_n.
4. The nonhomogeneous system $A\vec{x}=\vec{b}$ has a unique solution.
5. *A* is a product of elementary matrices.
6. $\det(A)\neq 0$

The following statements are equivalent:

1. An $n\times n$ matrix *A* is singular.
2. *A* is row equivalent to a matrix *B* that has a row of zeros.
3. The homogeneous system $A\vec{x}=0$ has a nontrivial solution.
4. $\det(A)=0$