Math 267 Fall 2009 Practice Finals NAME:

Do the following problems as indicated. Show all work.

1. Given two planes $P_1 : 3x - 2y - 2z + 1 = 0$ and $P_2 : x + 2y - 2z + 3 = 0$, find:
   a. The radian measure of the angle between P_1 and P_2
   b. Parametric equations of the line of intersection of P_1 and P_2
   c. Equation of the plane containing the line $\frac{x+1}{2} = \frac{y-2}{-3} = \frac{z-3}{2}$ and perpendicular to plane P_1.

2. Find $\vec{r}(t)$ if $\vec{r}'(t) = \sec^2 t\,\vec{i} + te^{-3t^2}\,\vec{j} + \sin t\,\vec{k}$ and $\vec{r}(0) = -2\vec{i} + \frac{1}{3}\vec{j} - \frac{1}{3}\vec{k}$.

3. Find an equation of the circle of curvature of the curve $y = \frac{1}{2}x^2$ at the point (-2, 2).

4. A spring gun at ground level fires a golf ball at an angle of 45°. The ball lands 30 feet away. Find:
   a. Vector function $\vec{r}(t)$
   b. The initial speed
   c. The maximum altitude attained
   d. Speed at impact.

5. a. Show $\lim_{(x,y)\to(0,0)} \frac{xy}{5x^2 - y^2}$ does not exist.

   b. Find $\lim_{(x,y)\to(-1,1)} \frac{x+y}{x^3 + y^3}$

6. Find $\frac{\partial^3 g}{\partial x \partial y \partial z}$ if $g(x, y, z) = xy\cos yz$.

7. The temperature at a point (x, y) on a rectangular metal plate is given by $T(x, y) = \frac{xy}{1 + x^2 + y^2}$ degrees Celsius.
   a. Find the rate of change of temperature at (1, 1) in te direction of $-4\vec{i} + 3\vec{j}$.
   b. An ant at (2, -2) wants to avoid walking in the direction in which the temperature rises most rapidly. Find a unit vector in that direction.
   c. Find a unit vector in the direction where there is no change in temperature.

8. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex at the plane $x + 2y + 3z = 6$.

9. Find local extrema and saddle points for the function $f(x,y) = x^2 y - 6y^2 - 3x^2$.

10. Evaluate the given integrals:
    a. $\int_0^1 \int_y^1 \cos\left(x^2\right) dx\, dy$
    b. $\iiint_Q y\, dV$ where Q is the region in the first octant that lies above the *xy*-plane, below the plane $z = x + 2$ and between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.
    c. $\int_C y\, dx + z\, dy + x\, dz$ where C is the graph of $x = \sin t,\ y = 2\sin t,\quad z = \sin^2 t\ ,\ 0 \le t \le \frac{\pi}{2}$

11. Find the surface area of the saddle $z = y^2 - x^2$ cut by the cylinder $x^2 + y^2 = 1$ that lies above the *xy*-plane.

12. Find the center of mass of the lamina that occupies the region in the first quadrant bounded by the parabola $x = 1 - y^2$ and the coordinate axes, if the density of the point (*x*, *y*) is given by $\delta(x,y) = y$.

13. <u>Set up the integral</u> (in cylindrical coordinates) for the volume of the solid *Q* above the *xy*-plane enclosed by the sphere $x^2 + y^2 + z^2 = 16$ and the cylinder $x^2 + y^2 = 4y$.

14. Use Green's Theorem to find the total work done in moving an object in the counterclockwise direction once around the circle $x^2 + y^2 - 2x = 0$ if the motion is caused by the force field $\vec{F}(x,y) = xy\,\vec{i} + (x^2 + y^2)\,\vec{j}$.

15. Show that the line integral $\int_C 2xe^{2y}\, dx + 2\left(x^2 e^{2y} + y\cot z\right) dy - y^2 \csc^2 z\, dz$ is independent of path. Then evaluate for a curve C from point $A\left(-2, 0, \frac{\pi}{4}\right)$ to point $B\left(1, 1, \frac{\pi}{6}\right)$.

16. Use Divergence Theorem to find the flux of the velocity field $\vec{F}(x,y,z) = xy^2\,\vec{i} + yz^2\,\vec{j} + zx^2\,\vec{k}$ across surface S, which is the region that lies between the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$, and between the planes $z = -1$ and $z = 2$.

17. Use Stoke's Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y,z) = y^2\,\vec{i} + 2x\,\vec{j} + 5y\,\vec{k}$ and C is the trace of the hemisphere $z = \sqrt{4 - x^2 - y^2}$ in the *xy*-plane.