Math 267 First Exam **Supplementary Review Problems**

1. Let $\vec{u} = \langle 1,3 \rangle$, $\vec{v} = \langle 2,1 \rangle$, $\vec{w} = \langle 4,-1 \rangle$. Find the vector $\vec{x}$ that satisfies $2\vec{u} - \vec{v} + \vec{x} = 7\vec{x} + \vec{w}$.

2. Use vectors to find the lengths of the diagonals of a parallelogram that has $\vec{i} + \vec{j}$ and $\vec{i} - 2\vec{j}$ as adjacent sides.

3. Let $\vec{r_1} = \langle x_1, y_1 \rangle$, $\vec{r_2} = \langle x_2, y_2 \rangle$, and $\vec{r} = \langle x, y \rangle$. Describe the set of all points (x, y) for which $\|\vec{r} - \vec{r_1}\| + \|\vec{r} - \vec{r_2}\| = k$, assuming that $k > \|\vec{r_2} - \vec{r_1}\|$.

4. Prove that $\|\vec{u} + \vec{v}\|^2 + \|\vec{u} - \vec{v}\|^2 = 2\|\vec{u}\|^2 + 2\|\vec{v}\|^2$ and interpret the result geometrically by translating it into a theorem about parallelograms.

5. The wind velocity $\vec{v_1}$ is 40 miles per hour from east to west while an airplane travels with air velocity $\vec{v_2}$ due 100 miles per hour due north. Find the true course of the airplane.

6. A force $\vec{F} = 4\vec{i} - 6\vec{j} + \vec{k}$ newtons is applied to a point that moves a distance of 15 meters in the direction of the vector $\vec{i} + \vec{j} + \vec{k}$. How much work is done?

7. Find the line through $(3,1,-2)$ that intersects and is perpendicular to the line

$$x = -1,\ y = -2 + t,\ z = -1 + t.$$

8. Find an equation of the plane that:

a. is perpendicular to $\vec{v} = (1,1,1)$ and passes through $(1,0,0)$.

b. is perpendicular to the line $\dfrac{x-3}{5} = \dfrac{z-1}{2}$, $y = -1$ and passes through $(5,-1,0)$.

c. passes through $(0,0,0), (2,0,-1)$ and $(0,4,-3)$.

d. passes through $(3,2,-1)$ and $(1,-1,2)$ and is parallel to the line

$$x = 1 + 3t,\ y = -1 + 2t,\ z = -2t.$$

9. Find the distance between the point $(6,1,0)$ and the plane passing through the origin that is perpendicular to $\vec{i} + 2\vec{j} + \vec{k}$.

10. Find the acute angle between the planes $x + 3y - 2z = 5$ and $2x - y + 4z = 7$.

11. Sketch the surface in 3-space.

a. $y = \sin x$

b. $z = 1 - y^2$

c. $4x^2 + 9z^2 = 36$

d. $2x + 3y = 6$

e. $4x + 3y + 2z = 12$

12. Show that for all values of θ and ϕ, the point

$$(a\sin\phi\cos\theta,\ a\sin\phi\sin\theta,\ a\cos\phi)$$

lies on the sphere $x^2 + y^2 + z^2 = a^2$.

Answers

1. $\left\langle -\frac{2}{3}, 1 \right\rangle$

2. $\sqrt{5}$ and 3

3. too long…ellipse with foci at (x_1, y_1) and (x_2, y_2) etc (let me know how Nick did this! ☺)

4. use $\vec{a} \bullet \vec{a} = \|a\|^2$; The sum of the squares of the diagonals of a parallelogram is equal to twice the sum of the squares of the sides

5. $\theta = \tan^{-1}\left(-\frac{2}{5}\right)$

6. $-5\sqrt{3}$ N-m

7. $x = -1 - 4t,\ y = -1 - 2t,\ z = 2t$

8a. $x + y + z - 1 = 0$

8b. $5x + 2z = 25$

8c. $2x + 3y + 4z = 0$

8d. $y + z = 1$

9. $8/\sqrt{6}$

10. $\theta = \cos^{-1}\frac{9}{\sqrt{14}\sqrt{21}}$