Math 267 Exam 1 Notes (Chapter 14)

Given $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$ vectors in V_2

$\vec{a} = \vec{b} \Leftrightarrow a_1 = b_1$ and $a_2 = b_2$ $\qquad \vec{a} + \vec{b} = \langle a_1, a_2 \rangle + \langle b_1, b_2 \rangle = \langle a_1 + b_1, a_2 + b_2 \rangle$ $\qquad \vec{0} = \langle 0, 0 \rangle$

$\|\vec{a}\| = \|\langle a_1, a_2 \rangle\| = \sqrt{a_1^2 + a_2^2}$ $\qquad c\vec{a} = c\langle a_1, a_2 \rangle = \langle ca_1, ca_2 \rangle$ $\qquad -\vec{a} = -\langle a_1, a_2 \rangle = \langle -a_1, -a_2 \rangle$

$\vec{a} - \vec{b} = \vec{a} + \left(-\vec{b}\right)$

Properties of Addition and Scalar Multiples of Vectors

(i) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ $\qquad$ **(iv)** $\vec{a} + \left(-\vec{a}\right) = \vec{0}$ $\qquad$ **(vii)** $(cd)\vec{a} = c\left(d\vec{a}\right) = d\left(c\vec{a}\right)$

(ii) $\vec{a} + \left(\vec{b} + \vec{c}\right) = \left(\vec{a} + \vec{b}\right) + \vec{c}$ $\qquad$ **(v)** $c\left(\vec{a} + \vec{b}\right) = c\vec{a} + c\vec{b}$ $\qquad$ **(viii)** $1\vec{a} = \vec{a}$

(iii) $\vec{a} + \vec{0} = \vec{a}$ $\qquad$ **(vi)** $(c+d)\vec{a} = c\vec{a} + d\vec{a}$ $\qquad$ **(ix)** $0\vec{a} = \vec{0} = c\vec{0}$

$\|c\vec{a}\| = |c|\,\|\vec{a}\|$

If $\vec{a} \neq \vec{0}$, then the **unit vector** $\vec{u}$ **that has the same direction as** $\vec{a}$ is $\vec{u} = \dfrac{\vec{a}}{\|\vec{a}\|}$.

distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is given by $d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

midpoint of line segment P_1P_2 is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}, \dfrac{z_1 + z_2}{2}\right)$

V_3 = the set of all vectors $\langle x, y, z \rangle$ where *x, y,* and *z* are real numbers

For $\vec{a} = \langle a_1, a_2, a_3 \rangle$, $\vec{b} = \langle b_1, b_2, b_3 \rangle$ in V_3,

$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$ $\qquad$ **unit vectors** $\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$, $\vec{k} = \langle 0, 0, 1 \rangle$

$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$ $\qquad$ $\vec{a} = \langle a_1, a_2, a_3 \rangle = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

$c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$

sphere with radius *r* and center $P_0(x_0, y_0, z_0)$ has equation $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$

Given $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$, the **dot product** $\vec{a} \bullet \vec{b}$ is defined by $\vec{a} \bullet \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

Properties of the dot product

(i) $\vec{a} \bullet \vec{a} = \|a\|^2$ $\qquad$ **(iii)** $\vec{a} \bullet \left(\vec{b} + \vec{c}\right) = \vec{a} \bullet \vec{b} + \vec{a} \bullet \vec{c}$ $\qquad$ **(v)** $\vec{0} \bullet \vec{a} = 0$

(ii) $\vec{a} \bullet \vec{b} = \vec{b} \bullet \vec{a}$ $\qquad$ **(iv)** $\left(c\vec{a}\right) \bullet \vec{b} = c\left(\vec{a} \bullet \vec{b}\right) = \vec{a} \bullet \left(c\vec{b}\right)$

If θ is the angle between nonzero vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \bullet \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos\theta$ or $\cos\theta = \dfrac{\vec{a} \bullet \vec{b}}{\|\vec{a}\|\|\vec{b}\|}$.

Two vectors $\vec{a}$ and $\vec{b}$ are **orthogonal** $\Leftrightarrow$ $\vec{a} \bullet \vec{b} = 0$

Cauchy-Schwarz Inequality $\quad |\vec{a} \bullet \vec{b}| \leq \|\vec{a}\|\|\vec{b}\|$

Triangle Inequality $\quad \|\vec{a} + \vec{b}\| \leq \|\vec{a}\| + \|\vec{b}\|$

Definition Let $\vec{a}$ and $\vec{b}$ be vectors in V_3 with $\vec{b} \neq \vec{0}$. The **component of** $\vec{a}$ **along** $\vec{b}$, is $\text{comp}_{\vec{b}}\vec{a} = \vec{a} \bullet \dfrac{\vec{b}}{\|\vec{b}\|}$

The **work done by a constant force** $\overrightarrow{PQ}$ as its point of application moves along the vector $\overrightarrow{PR}$ is $\overrightarrow{PQ} \bullet \overrightarrow{PR}$.

Given $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$. The **vector product** (or **cross product**) is

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$$

$\vec{a} \times \vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$.

If θ is the angle between nonzero vectors $\vec{a}$ and $\vec{b}$, then $\|\vec{a} \times \vec{b}\| = \|a\| \|b\| \sin\theta$

Two vectors $\vec{a}$ and $\vec{b}$ are **parallel** $\Leftrightarrow$ $\vec{a} \times \vec{b} = \vec{0}$.

Properties of $\vec{i}$, $\vec{j}$, $\vec{k}$:

$$\vec{i} \times \vec{j} = \vec{k} \qquad \vec{j} \times \vec{k} = \vec{i} \qquad \vec{k} \times \vec{i} = \vec{j}$$
$$\vec{j} \times \vec{i} = -\vec{k} \qquad \vec{k} \times \vec{j} = -\vec{i} \qquad \vec{i} \times \vec{k} = -\vec{j}$$
$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = \vec{0}$$

Properties of the vector product

(i) $\vec{b} \times \vec{a} = -\left(\vec{a} \times \vec{b}\right)$

(ii) $\left(m\vec{a}\right) \times \vec{b} = m\left(\vec{a} \times \vec{b}\right) = \vec{a} \times \left(m\vec{b}\right)$

(iii) $\vec{a} \times \left(\vec{b} + \vec{c}\right) = \left(\vec{a} \times \vec{b}\right) + \left(\vec{a} \times \vec{c}\right)$

(iv) $\left(\vec{a} + \vec{b}\right) \times \vec{c} = \left(\vec{a} \times \vec{c}\right) + \left(\vec{b} \times \vec{c}\right)$

(v) $\left(\vec{a} \times \vec{b}\right) \bullet \vec{c} = \vec{a} \bullet \left(\vec{b} \times \vec{c}\right)$

(vi) $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \bullet \vec{c}\right)\vec{b} - \left(\vec{a} \bullet \vec{b}\right)\vec{c}$ **triple vector product**

Parametric equations for the line through $P_1(x_1, y_1, z_1)$ parallel to $\vec{a} = \langle a_1, a_2, a_3 \rangle$ are

$$x = x_1 + a_1 t, \quad y = y_1 + a_2 t, \quad z = z_1 + a_3 t; \ t \text{ in } \mathbb{R}$$

Equation of the plane through $P_1(x_1, y_1, z_1)$ with normal vector $\vec{a} = \langle a_1, a_2, a_3 \rangle$ is

$$a_1(x - x_1) + a_2(y - y_1) + a_3(z - z_1) = 0.$$

Symmetric form for a line $\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{a_2} = \dfrac{z - z_1}{a_3}$

If one of the numbers a_1, a_2, or a_3 is zero, say $a_3 = 0$, then a symmetric form is $\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{a_2}$, $z = z_1$,

Distance from a point $P_0(x_0, y_0, z_0)$ **to the plane** $ax + by + cz + d = 0$

$$h = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Distance *d* **between two skew lines** l_1 **and** l_2, where P_1 and Q_1 are on l_1, and P_2 and Q_2 are on l_2

$$d = \frac{\left|\left(\overrightarrow{P_1Q_1} \times \overrightarrow{P_2Q_2}\right) \cdot \overrightarrow{P_1P_2}\right|}{\left\|\overrightarrow{P_1Q_1} \times \overrightarrow{P_2Q_2}\right\|}$$

Let *C* be a curve in a plane, and let *l* be a line that is not in a parallel plane. The set of all lines that are parallel to *l* and intersect *C* is a **cylinder**. The curve *C* in the plane is called a **directrix** for the cylinder, and each line through *C* parallel to *l* is a **ruling** of the cylinder. A **right circular cylinder** is obtained if *C* is a circle in a plane and *l* is a line perpendicular to the plane.