Math 266  Group Quiz #1  NAME:

Do the following as indicated. Show all work. No work, no points.

1. Differentiate each function
   a. \( f(x) = \ln \cos^2 2x \)

\[ g(x) = \frac{2}{e^x} \]

\[ h(x) = \log_6 \sqrt{x^2 - 2x} \]

2. Evaluate given integrals.
   a. \( \int \frac{1}{e^{-x}(4 + e^x)} \, dx \)

\[ \int x^2 \sec x^3 \, dx \]

\[ \int 10^{\cos x} \sin x \, dx \]
3. Use logarithmic differentiation to find \( y' \): 
\[
y = \frac{(4x+5)^3(x^2+2)^5}{(3x+7)^4}
\]

4. Let \( f \) be the function defined by \( f(x) = 2x^5 + x^3 + 3x \).
   a. Prove that \( f \) has an inverse function.

   b. Find the slope of the tangent line at the point \((6, 1)\) on the graph of \( y = f^{-1}(x) \).

5. Initially there were 100 mg of a radioactive substance present. After 6 hours, the mass decreased to 97 mg. If the rate of decay is proportional to the amount of the substance present at any time, find the following.
   a. A formula for the amount of the substance \( y = q(t) \) remaining at time \( t \).

   b. What amount remains after 24 hours?

   c. Determine the half-life of the substance