Math 265 Precalculus Review

Properties of Inequalities

1. $a > b$ and $b > c \Rightarrow a > c$

2. $a > b \Rightarrow a + c > b + c$

3. $a > b$ and $c > 0 \Rightarrow ac > bc$

4. $a > b$ and $c < 0 \Rightarrow ac < bc$

Absolute Value $\quad |a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$

Properties of Absolute Value $\quad b > 0$

1. $|a| < b \Leftrightarrow -b < a < b$

2. $|a| > b \Leftrightarrow a > b$ or $a < -b$

3. $|a| = b \Leftrightarrow a = b$ or $a = -b$

Interval Notation

$(a,b) = \{x : a < x < b\}$

$[a,b] = \{x : a \leq x \leq b\}$

$[a,b) = \{x : a \leq x < b\}$

$(a,b] = \{x : a < x \leq b\}$

$(a,\infty) = \{x : x > a\}$

$[a,\infty) = \{x : x \geq a\}$

$(-\infty,b) = \{x : x < b\}$

$(-\infty,b] = \{x : x \leq b\}$

$(-\infty,\infty) = \mathbb{R}$

Distance Formula between $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$

$$d(P_1,P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Symmetries of Graphs

1. y – axis: Substitution of $-x$ for x leads to the same equation.

2. x – axis: Substitution of $-y$ for y leads to the same equation.

3. origin: Substitution of both $-x$ and $-y$ for x and y leads to the same equation.

Center-radius form of the equation of a circle center $C(h,k)$ and radius r

$$(x-h)^2 + (y-k)^2 = r^2$$

If $r = 1$ then the circle is called a **unit circle**.

Lines and Slopes

slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$;

$m > 0 \Leftrightarrow$ line rises from left to right
$m < 0 \Leftrightarrow$ line falls from left to right
$m = 0 \Leftrightarrow$ line is *horizontal*
m undefined $\Leftrightarrow$ line is *vertical*
$m_1 = m_2 \Leftrightarrow$ lines are *parallel*
$m_1 m_2 = -1 \Leftrightarrow$ lines are *perpendicular*

Equations of Lines

point-slope form	$y - y_1 = m(x - x_1)$
slope-intercept form	$y = mx + b$
standard or general form	$ax + by = c$ or $ax + by + d = 0$
horizontal line	$y = k$, k constant
vertical line	$x = k$, k constant

Parabolas

$y = a(x-h)^2 + k$ vertex (h,k); opens up if $a > 0$, down if $a < 0$

$x = a(y-k)^2 + h$ vertex (h,k); opens right if $a > 0$, left if $a < 0$

$y = ax^2 + bx + c$ vertex (h,k); opens up if $a > 0$, down if $a < 0$

where $h = -\dfrac{b}{2a}$ and $k = \dfrac{4ac - b^2}{4a}$ (or simply substitute the value of h)

$x = ay^2 + by + c$ vertex (h,k); opens right if $a > 0$, left if $a < 0$

where $k = -\dfrac{b}{2a}$ and $h = \dfrac{4ac - b^2}{4a}$ (or simply substitute the value of k)

Functions

A **function** f from a set D to a set E, $f : D \to E$, is a correspondence that assigns to each element x of D exactly one element y of E. The set D is the **domain** of the function. The **range** of f is the subset of E consisting of all possible function values $f(x)$ for x in D.

A function is usually defined by giving the formula or rule for finding $f(x)$. The domain is then the set of all values of x for which $f(x)$ is real. f is said to be **defined** at x if $f(x)$ is real or $f(x)$ exists, i.e., x is in the domain of f; f is said to be **undefined** at x if x is not in the domain of f.

In the notation $y = f(x)$, x is called the **independent variable** and y is called the **dependent variable**.

Graph of a function is the graph of the equation $y = f(x)$ for x in the domain of f.

Vertical Line Test: Every vertical line intersects the graph of a function in *at most one point*.

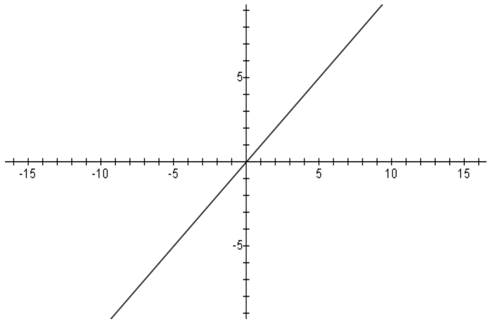
zeros of a function: solutions of the equation $f(x) = 0$; the x-intercepts of the graph

even function: if $f(-x) = f(x)\ \forall x \in D_f$; graph is symmetric wrt y-axis

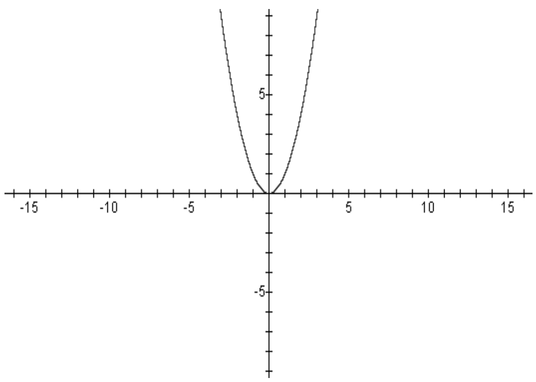
odd function: if $f(-x) = -f(x)\ \forall x \in D_f$; graph is symmetric wrt origin

Basic Graphs

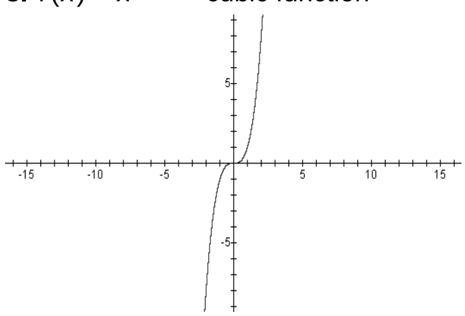
1. $f(x) = x$ linear function (identity function)



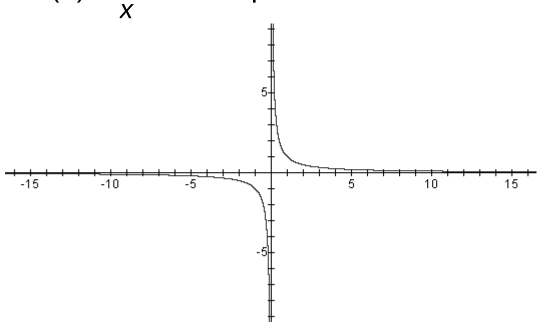
2. $f(x) = x^2$ quadratic function



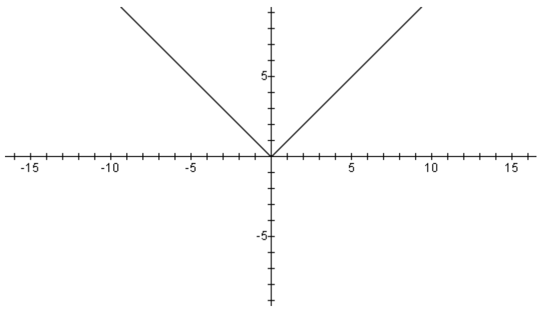
3. $f(x) = x^3$ cubic function



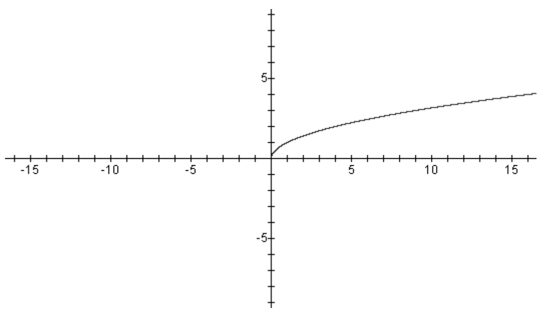
4. $f(x) = \dfrac{1}{x}$ reciprocal function



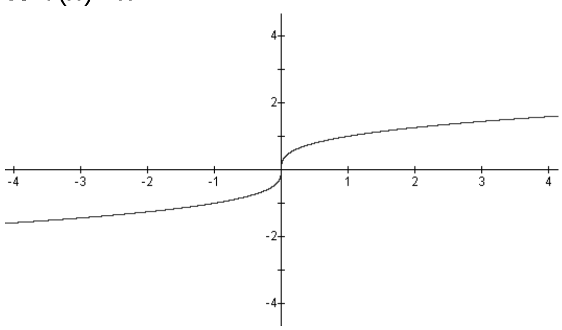
5. $f(x) = |x|$ absolute value function



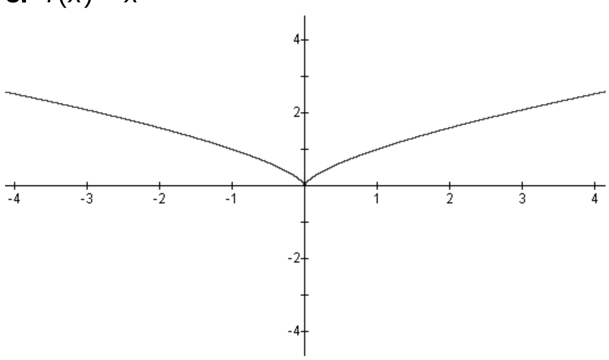
6. $f(x) = \sqrt{x}$ square root function



7. $f(x) = x^{1/3}$



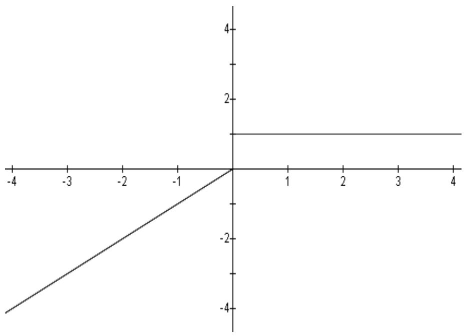
8. $f(x) = x^{2/3}$



Piecewise-defined functions

- functions that are defined by more than one expression

example: $f(x) = \begin{cases} x & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$



greatest integer function $f(x) = [\![x]\!]$ is the greatest integer n such that $n \leq x$;
on the coordinate line, n is the first integer to the *left of* (or *equal to*) x.

Transformations of Graphs

1. vertical shifts - result when a positive constant c is added to or subtracted from $f(x)$

ex. $y = x^2 + 2$ basic parabola shifted 2 units down

$y = x^2 - 4$ basic parabola shifted 4 units down

2. horizontal shifts

ex. $y = (x - 3)^2$ basic parabola shifted 3 units to the right

$y = (x + 5)^2$ basic parabola shifted 5 units to the left

3. reflections

ex. $y = -x^2$ reflection of the basic parabola along the x-axis

4. compressions/expansions

ex. $y = \frac{1}{2}x^2$ expanded basic parabola

$y = 3x^2$ compressed basic parabola

5. combination of the above

ex. $y = -3(x + 1)^2 + 5$

Operations on Functions

1. sum $f + g$: $(f + g)(x) = f(x) + g(x)$

2. difference $f - g$: $(f - g)(x) = f(x) - g(x)$

3. product fg: $(fg)(x) = f(x)g(x)$

4. quotient f/g: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

The domain of $f+g$, $f-g$, fg, and f/g is the *intersection* of the domains of *f* and *g*; for f/g, the domain is the set of all *x* in the intersection for which $g(x)\neq 0$.

Composition of Functions

$$(f\circ g)(x)=f(g(x))$$

The domain of $f\circ g$ is the set of all *x* in the domain of *g* such that *g(x)* is in the domain of *f*.

Further Classification of Functions

polynomial $f(x)=a_n x^n + a_{n-1}x^{n-1}+\cdots+a_1 x+a_0$

where $a_0, a_1, \ldots, a_n$ are real and the exponents are nonnegative integers

If $a_n \neq 0$, then *f* has **degree** *n*.

degree 0: $f(x)=a$ **constant function**

degree 1: $f(x)=ax+b$ **linear function**

degree 2: $f(x)=ax^2+bx+c$ **quadratic function**

rational $f(x)=\dfrac{p(x)}{g(x)}$, where *p* and *g* are polynomial functions

algebraic can be expressed in terms of sums, differences, products, quotients, or rational powers of polynomials

transcendental not algebraic
ex. trigonometric, exponential, logarithmic

Trigonometry

<u>Radians and Degrees</u> 2π radians $=180^\circ$

radian to degree: multiply by $180/\pi$

degree to radian: multiply by $\pi/180$

<u>Length of a Circular Arc</u> $s=r\theta$

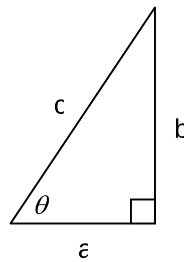
where s is the length of the arc
r radius of the circle
θ *radian* measure of the central angle subtended by the arc

<u>Area of a Circular Sector</u> $A=\frac{1}{2}r^2\theta$

where θ is the *radian* measure of a central angle
r radius of the circle

Trigonometric Functions

A. of an acute angle θ:

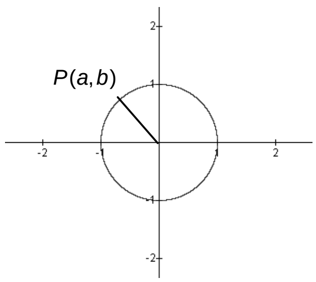


$$\sin\theta = \frac{b}{c} \qquad \csc\theta = \frac{c}{b}$$

$$\cos\theta = \frac{a}{c} \qquad \sec\theta = \frac{c}{a}$$

$$\tan\theta = \frac{b}{a} \qquad \cot\theta = \frac{a}{b}$$

B. of any angle θ



$$x^2 + y^2 = r^2$$

$$\sin\theta = \frac{b}{r} \qquad \csc\theta = \frac{r}{b}$$

$$\cos\theta = \frac{a}{r} \qquad \sec\theta = \frac{r}{a}$$

$$\tan\theta = \frac{b}{a} \qquad \cot\theta = \frac{a}{b}$$

C. of a real number

value of a trig function of a real number x = value of trig function at an angle of x radians

Fundamental Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta = 1$$

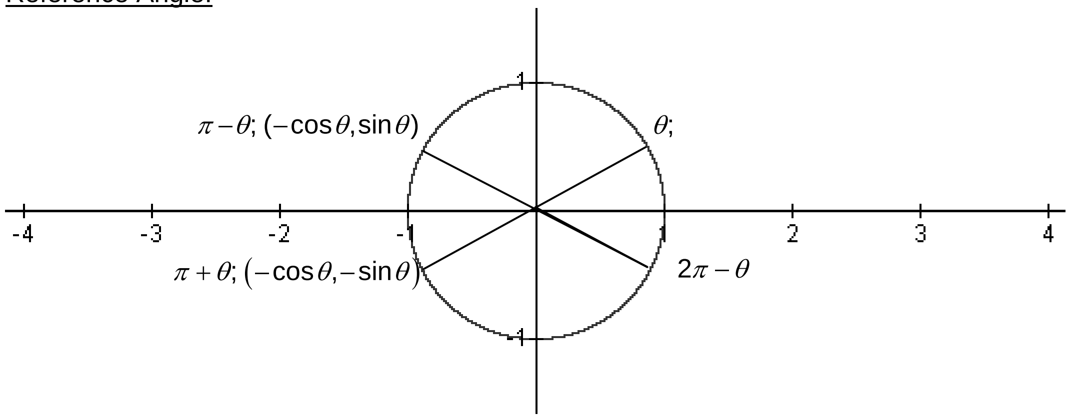
$$\sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} \qquad 1 + \tan^2\theta = \sec^2\theta$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad 1 + \cot^2\theta = \csc^2\theta$$

Special Values of the Trigonometric Functions

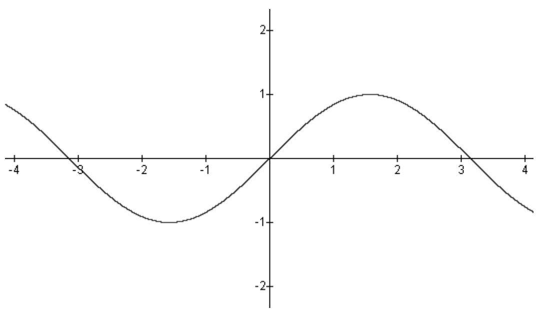
θ **(radians)**	θ **(degrees)**	$\sin\theta$	$\cos\theta$	$\tan\theta$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Reference Angle:

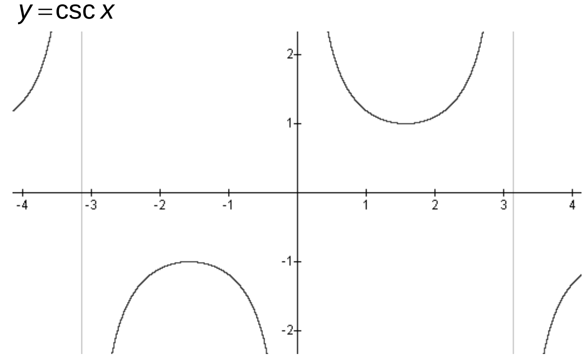


Graphs of the Trigonometric Functions

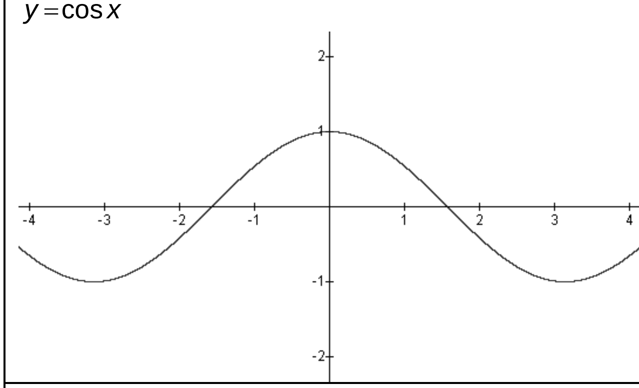
$y=\sin x$



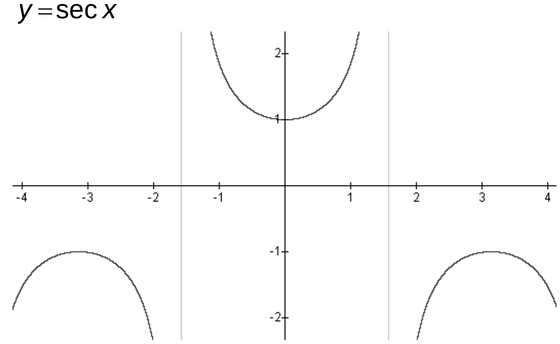
$y=\csc x$



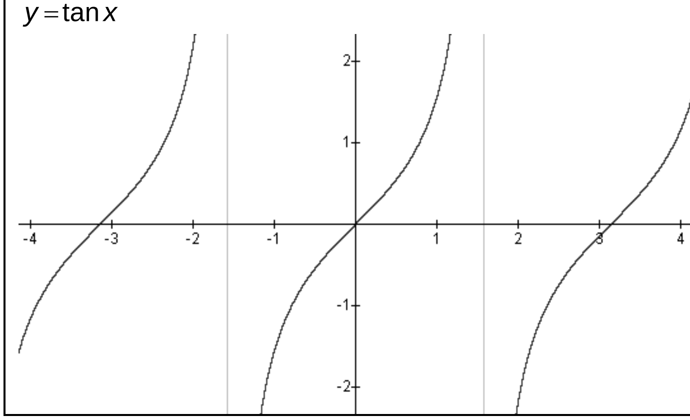
$y=\cos x$



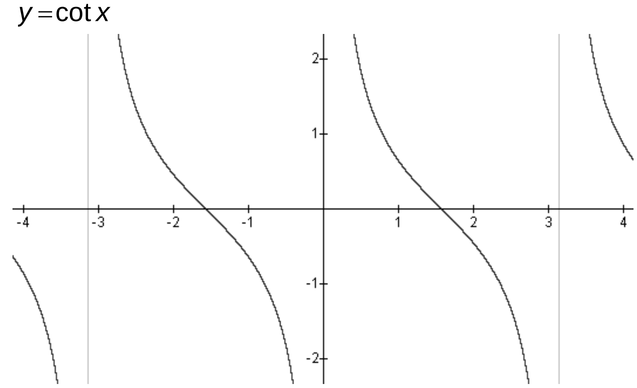
$y=\sec x$



$y=\tan x$



$y=\cot x$



Other Formulas

1. *formulas for negatives*

$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$

$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot u$

2. *addition and subtraction formulas*

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\tan(u \pm v) = \dfrac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$

3. *double-angle formulas*

$\sin 2u = 2 \sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2\tan u}{1 - \tan^2 u}$

4. *half-angle formulas*

$\sin^2 u = \dfrac{1 - \cos 2u}{2}$

$\cos^2 u = \dfrac{1 + \cos 2u}{2}$