

Math 265 Final Exam Review

Evaluate the following limits.

1. $\lim_{x \rightarrow 3} \frac{x + \sqrt{x+1}}{x^2 - 1}$

4. $\lim_{w \rightarrow 1} \frac{\sqrt{w+3} - 2}{w - 1}$

2. $\lim_{t \rightarrow -2} \frac{3t^2 + 5t - 2}{t^2 - 3t - 10}$

5. $\lim_{\theta \rightarrow 0} \frac{\sin 3\theta}{\sin 5\theta}$

3. $\lim_{y \rightarrow 5^-} \frac{y^2 - 5y}{|y - 5|}$

Prove the following using the definition of limit.

6. $\lim_{x \rightarrow 4} (3x - 5) = 7$

7. Find the values of the constants a and b that make the function f continuous on $(-\infty, \infty)$.

$$f(x) = \begin{cases} 2 & \text{if } x < 0 \\ ax^2 + b & \text{if } 0 \leq x < 3 \\ 2x & \text{if } x \geq 3 \end{cases}$$

Use the definition of the derivative to find f' .

8. $f(x) = x^2 - 4$

Use differentiation formulas to find f' . Do not simplify the answer.

9. $f(x) = 6x^2 - \frac{5}{x} + \frac{2}{\sqrt[3]{x^2}}$

10. $f(w) = \sqrt{\frac{2w+5}{7w-9}}$

11. $f(t) = \csc^3 3t \cot^2 3t$

Use implicit differentiation to find y' .

12. $\sqrt{xy} = x^2 + y^2$

13. Prove that there is no line through $(-1, 2)$ that is tangent to the graph of $f(x) = x^2 + 3x$.

14. The number of dollars in a small machine shop's weekly profit as a function of the number of custom items is given by

$$P(x) = 150x^2 - 2.3x^3 - 1200.$$

Find the marginal profit for production of 15 items.

15. In a high wind, the top of a tall building sways back and forth with a displacement x given by

$$x(t) = 1.7 \cos(0.21t + 3.7),$$

where time is measured in seconds and displacement in meters. Find the velocity and acceleration of the top of the building when $t = 1$.

16. The pressure P and volume V of a sample of gas undergoing an isothermal expansion are related by

$$P = \frac{2.5}{V},$$

where pressure is measured in pascals and volume in cm^3 . If the volume is increasing at a rate of $50 \text{ cm}^3/\text{sec}$, find the rate at which pressure is changing when the volume is 10 cm^3 .

17. Use limits to find all the asymptotes of the graph of

$$f(x) = \frac{x^2}{x-1}.$$

Sketch the graph.

18. Draw a sketch of the graph of

$$f(x) = x^3 - 3x^2 + 1$$

by first finding the following:

- any local extrema of f ,
- the intervals where f is increasing/decreasing,
- the intervals where the graph of f is concave up/down, and
- any points of inflection.

Indicate the local extrema and point of inflection (if any) on the sketch.

19. The radius of the earth averages 3960 miles. Assuming that the earth is a sphere, use differentials to approximate the volume of the earth's outer layer or crust, if its thickness averages 15 miles.
20. Find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius a , if two vertices lie on the diameter.