**Math 265** **Exam 3 Review Questions**

**1.** Find the critical numbers of $f(x) = \dfrac{1+\cos x}{1-\cos x}$.

**2.** Find the absolute extrema of $f(x) = 1 - x^{2/3}$ on $[-1,8]$.

**3.** Given $f(x) = 6 + 4x^3 - 3x^4$ .

**a.** Find the critical numbers of *f*, if any.
**b.** Use the First Derivative Test to determine the local extrema of *f*, if any.
**c.** Determine the intervals where *f* is increasing or decreasing.
**d.** Determine the points of inflection, if any.
**e.** Determine the concavity of the graph.
**f.** Sketch the graph.

**4.** Given $f(x) = \dfrac{x^2}{x^2-4}$ with derivatives $f'(x) = \dfrac{-8x}{\left(x^2-4\right)^2}$ and $f''(x) = \dfrac{24x^2+32}{\left(x^2-4\right)^3}$.

**a.** Find the critical numbers of *f*, if any.
**b.** Use the Second Derivative Test to determine the local extrema of *f*, if any.
**c.** Determine the points of inflection, if any.
**d.** Determine the concavity of the graph.
**e.** Find all the asymptotes of the graph, if any.
**f.** Sketch the graph.

**5.** Show that the rectangle with area 100 and minimum perimeter is a square.

**6.** Given $y = x^4 - 2x^3 + 4x - 3$.

**a.** At which point on the graph is the slope of the tangent line a local maximum?
**b.** At which point is the slope decreasing most rapidly?

**7.** A real estate developer is contemplating the construction of a luxury condominium building. Profit per unit is expected to be $200,000 if 20 units are built, but will decrease by $4000 for each additional unit built on the site. How many units should be built in order to maximize total profit?

**8.** A manufacturer estimates that his weekly cost of production is given by the formula

$$C = 8000 + 7x + 0.0001x^2,$$

where *x* is the number of articles manufactured. If he sets the selling price at *y*, he estimates that each week he can sell

$$x = 11000 - 500y$$

articles. How many articles should he try to manufacture each week and what selling price should he set for each article if he wants to maximize his profit?

**9.** The position function of a particle moving on a coordinate line is given by

$$s(t) = 2t^3 - 21t^2 + 60t + 3.$$

Answer the following for $t \geq 0$.

**a.** When is the particle momentarily at rest?
**b.** When is the particle moving to the left? When is it moving to the right?
**c.** When does the particle speed up? When does it slow down?
**d.** Draw a diagram describing the motion of the particle.

**10.** A projectile is fired directly upward with an initial velocity of 144 ft/sec and its height (in feet) above the ground is given by

$$s(t) = 144t - 16t^2 .$$

Find

**a.** the velocity and acceleration after *t* seconds,

**b.** the maximum height, and

**c.** the duration of the flight.

**11.** Use the fact that

$$D_x\left(x^6 - 2x^2 + x\right) = 6x^5 - 4x + 1$$

to show that the equation $6x^5 - 4x + 1 = 0$ has at least one solution in the interval $(0,1)$.

**12.** The geometric mean of two positive numbers *a* and *b* is the number $\sqrt{ab}$. Show that the value *c* in the conclusion of the Mean Value Theorem for $f(x) = \frac{1}{x}$ on the interval of positive numbers $[a, b]$ is $c = \sqrt{ab}$.