Math 265 Exam #2 Review NAME: _____

Do the following as indicated.

1. Find the derivative of each function. Simplify result but don't overdo it.

   a. $f(x)=\dfrac{x^4-2x^2+8}{4\sqrt[3]{x^2}}$

   b. $g(x)=x^3\sec\frac{1}{x^3}$

   c. $h(x)=\dfrac{2x}{\sqrt{x^2+3x-4}}$

   d. $F(x)=\dfrac{\tan\sqrt{x}}{\sqrt{x}}$

   e. $G(x)=\dfrac{\left(x^3+6x+1\right)^{5/3}}{5}$

   f. $H(x)=x^3-x^2\cos x+2x\sin x+2\cos x$

2. Evaluate each limit.

   a. $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{\sin 3x}$

   b. $\lim\limits_{x\to 0}\dfrac{2x^3}{\tan^3 2x}$

3. Find $\dfrac{dy}{dx}$ implicitly given $x^2y+\csc x^2y=4$.

4. Find $\dfrac{d^3y}{dx^3}$ if $y=\dfrac{1}{(1-x)^2}$.

5. a. Find an equation of the normal line to the curve $y=\dfrac{4}{1+x^2}$ at the point $(-1,2)$.

   b. Find an equation of the tangent line to the curve $2x^2+y^2=12$ that is parallel to the line $2x+y=4$.

6. A ball rolls down an inclined plane such that the distance (in cm) that it rolls in t seconds is given by $s(t)=2t^3+3t^2+4$ for $0\le t\le 3$.

   a. Find the velocity of the ball at $t=2$.

   b. At what time is the velocity $30\,cm/\sec$?

7. Let $y=(1+x)^{15}$, $x=0$, $\Delta x=.01$. Find:

   a. Δy b. dy c. $\Delta y-dy$

8. A metal rod 15 cm long and 5 cm in diameter is to be covered (except for the ends) with insulation that is .01 cm thick. Use differentials to estimate the volume of insulation.

9. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at the rate of $2\,cm^3/\sec$, how fast is the level rising when the water is 5 cm deep?

10. The radius of size 5 soccer balls is estimated to be 4.5 inches with a maximum error in measurement of $\pm .05$ in. Use differentials to estimate the maximum error in the calculated volume. Approximate the average error and percentage error.

11. Two boats are racing, each with constant speed toward a finish marker. Boat A is sailing from the south at 13 mph, and boat B is approaching from the east at another speed. When the boats are 16 miles apart, the distance between them is decreasing at the rate of 17 mph. At this same instant, the boats are at the same distance from the finish marker. Find the speed of boat B. Which boat will win the race?

ANSWERS:

1. a. $f'(x)=\dfrac{5x^4-4x^2-8}{6\sqrt[3]{x^5}}$ or $f'(x)=\dfrac{5}{6}x^{\frac{7}{3}}-\dfrac{2}{3}x^{\frac{1}{3}}-\dfrac{4}{3x^{\frac{5}{3}}}$

   b. $g'(x)=\dfrac{3}{x}\sec\left(\frac{1}{x^3}\right)\left[x^3-\tan\left(\frac{1}{x^3}\right)\right]$

   c. $h'(x)=\dfrac{3x-8}{\sqrt{\left(x^2+3x+4\right)^3}}$

   d. $F'(x)=\dfrac{\sqrt{x}\sec^2\sqrt{x}-\tan\sqrt{x}}{2x\sqrt{x}}$

   e. $G'(x)=\left(x^2+2\right)\left(x^3+6x+1\right)^{\frac{2}{3}}$

   f. $H'(x)=x^2\left(3-\sin x\right)$

2. a. 0 b. ¼

3. $y'=-\dfrac{2y}{x}$

4. $\dfrac{d^3y}{dx^3}=\dfrac{24}{(1-x)^5}$

5. a. $x+2y=3$ b. $y=-2x+6$ and $y=-2x-6$

6. a. $v(2)=36$ cm/sec b. $t=\dfrac{-1+\sqrt{21}}{2}\approx 1.8$ sec

7. a. $\Delta y=0.160969$ b. $dy=0.15$ c. $\Delta y-dy=0.010969$

8. $\Delta V\approx dV=0.75\pi\approx 2.356$ cu cm

9. $\dfrac{dh}{dt}=\dfrac{8}{9\pi}$ cm/sec ≈ 0.28 cm/sec

10. $\Delta V\approx dV=\pm 4.05\pi$ cu in $\approx\pm 12.72$ cu in ; $\dfrac{dV}{V}=\pm\dfrac{1}{30}\approx\pm 0.033=\pm 3.3\%$

11. Speed of boat B is 11 mph. Boat A will win.