

Math 265 Review Problems for the First Exam

1. Express the radius of a sphere as a function of the sphere's surface area. Then express the surface area as a function of the volume.
2. A point P in the first quadrant lies on the parabola $y = x^2$. Express the coordinates of P as functions of the angle of inclination of the line joining P to the origin.
3. A hot-air balloon rising straight up from a level field is tracked by a ranger finder located 500 ft from the point of lift-off. Express the balloon's height as a function of the angle of the line from the ranger finder to the balloon makes with the ground.
4. Discuss the continuity of

$$f(x) = \begin{cases} x^2 - 1, & -1 \leq x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$

5. For what value of a is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

continuous at every x ?

6. A parking garage charges \$5 for the first half-hour, \$3 for each additional half-hour (or fraction thereof) up to two hours, and \$1 for each half-hour after two hours. If t is the number of hours, and $0 \leq t \leq 3$,
 - (a) express the number of dollars in the total parking charge as a function of t .
 - (b) Sketch the graph of this function.
 - (c) At what numbers in the open interval $(0, 3)$ is the function discontinuous?

Use the limit rules to evaluate the limits or to show that they do not exist.

7. $\lim_{x \rightarrow 3} \frac{2x}{x^2 - x - 3}$

11. $\lim_{x \rightarrow -2} \frac{x^2 - 4}{|x + 2|}$

8. $\lim_{x \rightarrow -2} \frac{x+2}{x^2 + x - 2}$

12. $\lim_{x \rightarrow 5^+} \frac{25 - x^2}{x^2 - 10x + 25}$

9. $\lim_{x \rightarrow 0} \frac{4x - x^3}{3x + x^2}$

13. $\lim_{x \rightarrow \infty} \frac{2 + \sqrt{x}}{2 - \sqrt{x}}$

10. $\lim_{x \rightarrow -4} \frac{\frac{1}{\sqrt{13+x}} - \frac{1}{3}}{x+4}$

14. $\lim_{x \rightarrow \infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$

Sketch the graph of a function that satisfies the given conditions.

15. $f(0) = 0$, $f(1) = 2$, $f(-1) = -2$, $\lim_{x \rightarrow -\infty} f(x) = -1$, and $\lim_{x \rightarrow \infty} f(x) = 1$

16. $f(0) = 0$, $f(1) = 2$, $f(2) = 1$, $f(-1) = 0$, $\lim_{x \rightarrow \infty} f(x) = 0$, $\lim_{x \rightarrow -\infty} f(x) = 1$, $\lim_{x \rightarrow 0^-} f(x) = -\infty$, and $\lim_{x \rightarrow 0^+} f(x) = \infty$