

1. a. $\csc \theta = -\frac{3}{2}$, $\cos \theta = \frac{\sqrt{5}}{3}$, $\sec \theta = \frac{3\sqrt{5}}{5}$, $\tan \theta = -\frac{2\sqrt{5}}{5}$, $\cot \theta = -\frac{\sqrt{5}}{2}$
 b. $\cot \theta = \frac{7}{24}$, $\sin \theta = -\frac{24}{25}$, $\csc \theta = -\frac{25}{24}$, $\cos \theta = -\frac{7}{25}$, $\sec \theta = -\frac{25}{7}$
 c. $\cos \theta = -\frac{4}{5}$, $\sin \theta = \frac{3}{5}$, $\csc \theta = \frac{5}{3}$, $\tan \theta = -\frac{3}{4}$, $\cot \theta = -\frac{4}{3}$
 d. $\sin \theta = \frac{12}{13}$, $\csc \theta = \frac{13}{12}$, $\sec \theta = \frac{13}{5}$, $\tan \theta = \frac{12}{5}$, $\cot \theta = \frac{5}{12}$
 e. $\tan \theta = -\frac{1}{7}$, $\sin \theta = -\frac{\sqrt{2}}{10}$, $\csc \theta = -5\sqrt{2}$, $\cos \theta = \frac{7\sqrt{2}}{10}$, $\sec \theta = \frac{5\sqrt{2}}{7}$
 f. $\sin \theta = \frac{1}{9}$, $\cos \theta = -\frac{4\sqrt{5}}{9}$, $\sec \theta = -\frac{9\sqrt{5}}{20}$, $\tan \theta = -\frac{\sqrt{5}}{20}$, $\cot \theta = -4\sqrt{5}$

2. a. 3.56 b. -18 c. $-\frac{\sqrt{5}}{2}$ d. -5.5 e. .21

3. There are several ways one can verify identities. One way is given for each.

$$\text{a. } \frac{\tan \theta}{\sec \theta} = \frac{\frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta}} = \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{1} = \sin \theta$$

$$\text{b. } \frac{\sin^2 \theta}{\cos \theta} = \frac{1 - \cos^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} - \frac{\cos^2 \theta}{\cos \theta} = \sec \theta - \cos \theta$$

$$\begin{aligned} \text{c. } \tan^2 \beta \sin^2 \beta &= \tan^2 \beta (1 - \cos^2 \beta) = \tan^2 \beta - \tan^2 \beta \cos^2 \beta = \tan^2 \beta - \frac{\sin^2 \beta}{\cos^2 \beta} \cdot \cos^2 \beta \\ &= \tan^2 \beta - \sin^2 \beta = \tan^2 \beta - (1 - \cos^2 \beta) = \tan^2 \beta - 1 + \cos^2 \beta \end{aligned}$$

$$\begin{aligned} \text{d. } \frac{1}{\sec \beta - \tan \beta} &= \frac{1}{\frac{1}{\cos \beta} - \frac{\sin \beta}{\cos \beta}} = \frac{1}{\frac{1 - \sin \beta}{\cos \beta}} = \frac{\cos \beta}{1 - \sin \beta} = \frac{\cos \beta}{1 - \sin \beta} \cdot \frac{1 + \sin \beta}{1 + \sin \beta} \\ &= \frac{\cos \beta + \cos \beta \sin \beta}{1 - \sin^2 \beta} = \frac{\cos \beta + \cos \beta \sin \beta}{\cos^2 \beta} = \frac{\cos \beta}{\cos^2 \beta} + \frac{\cos \beta \sin \beta}{\cos^2 \beta} \\ &= \frac{1}{\cos \beta} + \frac{\sin \beta}{\cos \beta} = \sec \beta + \tan \beta \end{aligned}$$

$$3. \quad e. \quad \frac{\csc \theta + \cot \theta}{\tan \theta + \sin \theta} = \frac{\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \sin \theta} = \frac{\frac{1 + \cos \theta}{\sin \theta}}{\frac{\sin \theta + \sin \theta \cos \theta}{\cos \theta}}$$

$$= \frac{1 + \cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta (1 + \cos \theta)} = \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta} = \cot \theta \csc \theta$$

$$f. \quad \sec(\pi - x) = \frac{1}{\cos(\pi - x)} = \frac{1}{\cos \pi \cos x + \sin \pi \sin x} = \frac{1}{-\cos x} = -\sec x$$

$$g. \quad \frac{\sin(x + y)}{\cos x \cos y} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y} = \frac{\sin x \cos y}{\cos x \cos y} + \frac{\cos x \sin y}{\cos x \cos y} = \tan x + \tan y$$

$$h. \quad \sin 4\theta = 2 \sin 2\theta \cos 2\theta = 2(2 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta) = 4 \sin \theta \cos \theta - 8 \sin^3 \theta \cos \theta$$

$$i. \quad \frac{\tan(A + B) - \tan B}{1 + \tan(A + B) \tan B} = \tan[(A + B) - B] = \tan A$$

$$j. \quad \frac{\cot \alpha - \tan \alpha}{\cot \alpha + \tan \alpha} = \frac{\frac{\cos \alpha}{\sin \alpha} - \frac{\sin \alpha}{\cos \alpha}}{\frac{\cos \alpha}{\sin \alpha} + \frac{\sin \alpha}{\cos \alpha}} = \frac{\frac{\cos^2 \alpha - \sin^2 \alpha}{\sin \alpha \cos \alpha}}{\frac{\cos^2 \alpha + \sin^2 \alpha}{\sin \alpha \cos \alpha}}$$

$$= \frac{\cos^2 \alpha - \sin^2 \alpha}{\sin \alpha \cos \alpha} \cdot \frac{\sin \alpha \cos \alpha}{1} = \cos 2\alpha$$

$$4. \quad a. \quad \frac{\sqrt{2} + \sqrt{6}}{4} \qquad b. \quad \frac{\sqrt{6} + \sqrt{2}}{4} \qquad c. \quad \frac{\sqrt{3} + 3}{3 - \sqrt{3}} \text{ or } 2 + \sqrt{3}$$

$$d. \quad \frac{\sqrt{6} - \sqrt{2}}{4} \qquad e. \quad \frac{\sqrt{6} - \sqrt{2}}{4} \qquad f. \quad \frac{-\sqrt{3} + 1}{1 + \sqrt{3}} \text{ or } -2 + \sqrt{3}$$

$$5. \quad a. \quad \sin(x - y) = \frac{-2\sqrt{30} - 2}{15}, \quad \cos(x + y) = \frac{\sqrt{5} + 4\sqrt{6}}{15},$$

$$\tan(x - y) = \frac{-10\sqrt{6} - 2\sqrt{5}}{5 - 4\sqrt{30}} = \frac{18\sqrt{6} + 50\sqrt{5}}{91}, \quad x - y \text{ in quadrant III}$$

$$b. \quad \sin(x - y) = \frac{323}{325}, \quad \cos(x + y) = \frac{204}{325}, \quad \tan(x - y) = \frac{323}{204}, \quad x - y \text{ in quadrant I}$$

$$c. \quad \sin(x - y) = \frac{-2\sqrt{3} - \sqrt{42}}{12}, \quad \cos(x + y) = \frac{-2\sqrt{21} - \sqrt{6}}{12},$$

$$\tan(x - y) = \frac{2\sqrt{7} + 7\sqrt{2}}{14 - \sqrt{14}} = \frac{3\sqrt{7} + 8\sqrt{2}}{13}, \quad x - y \text{ in quadrant III}$$

6. a. False b. True c. False d. False e. True

7. a. $\sin A = -\frac{\sqrt{7}}{4}$ b. $\tan 2x = -\frac{5}{12}$ c. $\cos \frac{A}{2} = \frac{\sqrt{2}}{4}$

d. $\tan \frac{\theta}{2} = \frac{-\sqrt{10} - \sqrt{6}}{2}$ e. $\sin \frac{B}{2} = \frac{\sqrt{50 - 20\sqrt{6}}}{10}$

8. a. $\tan x$ b. $\tan \frac{x}{2}$ c. $\cos 40^\circ$ d. $\cos 70^\circ$ e. $\sin 10\theta$ f. 1

9. a. $\frac{\sqrt{2-\sqrt{2}}}{2}$ b. $2+\sqrt{3}$ c. $\frac{\sqrt{2+\sqrt{3}}}{2}$ d. $\frac{\sqrt{2-\sqrt{3}}}{2}$ e. $\sqrt{2}+1$ f. $-\sqrt{2}+1$

10. a. $-\frac{\pi}{4}$ b. $\frac{\pi}{6}$ c. $\frac{\pi}{2}$ d. $\frac{5\pi}{6}$ e. $\frac{\pi}{4}$ f. $\frac{\pi}{3}$

11. a. 144.74° b. 88.09° c. 7.09° d. 54.74° e. 164.69° f. -54.34°

12. a. $\frac{\sqrt{15}}{4}$ b. $\frac{3\sqrt{73}}{73}$ c. $\frac{24}{7}$ d. $\frac{63}{65}$ e. $\frac{4\sqrt{77}}{81}$ f. $\frac{33}{56}$

13. a. $x = \frac{1}{2} \left[1 + \cot^{-1} \left(\frac{y}{3} \right) \right]$ b. $x = -5 + \frac{1}{3} \sin^{-1} \left(\frac{y+3}{2} \right)$ c. $x = -1$

d. $x = \frac{24}{7}$ e. $x = \frac{2\sqrt{5}}{5}$

14. a. $x = 0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ b. $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ c. $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

d. $\theta = 15^\circ, 45^\circ, 75^\circ, 105^\circ, 135^\circ, 165^\circ, 195^\circ, 225^\circ, 255^\circ, 285^\circ, 315^\circ, 345^\circ$

e. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ f. $\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ, 45^\circ, 135^\circ, 225^\circ, 315^\circ$

15. a. $\sqrt{1-x^2}$ b. $\frac{1}{x}$ c. $\frac{2x}{1+x^2}$