

Do the following as indicated. Show your work.

- Use the fundamental identities to find exact values of the remaining five trigonometric functions of θ .
 - $\sin \theta = -\frac{2}{3}$, $\cos \theta > 0$
 - $\tan \theta = \frac{24}{7}$, $\csc \theta < 0$
 - $\sec \theta = -\frac{5}{4}$, $\cot \theta < 0$
 - $\cos \theta = \frac{5}{13}$, θ in quadrant I
 - $\cot \theta = -7$, θ in quadrant IV
 - $\csc \theta = 9$, θ in quadrant II
- Fill in the blanks.
 - If $\tan s = -3.56$ then $\tan(-s) = \underline{\hspace{2cm}}$
 - If $\sec s = -18$ then $\sec(-s) = \underline{\hspace{2cm}}$
 - If $\sin \theta = -\frac{2}{3}$ and $\cos \theta = -\frac{\sqrt{5}}{3}$ then $\cot(-\theta) = \underline{\hspace{2cm}}$
 - If $\csc x = 5.5$ then $\csc(-x) = \underline{\hspace{2cm}}$
 - If $\cos x = .21$ then $\cos(-x) = \underline{\hspace{2cm}}$
- Verify each trigonometric identity.
 - $\frac{\tan \theta}{\sec \theta} = \sin \theta$
 - $\frac{\sin^2 \theta}{\cos \theta} = \sec \theta - \cos \theta$
 - $\tan^2 \beta \sin^2 \beta = \tan^2 \beta + \cos^2 \beta - 1$
 - $\frac{1}{\sec \beta - \tan \beta} = \sec \beta + \tan \beta$
 - $\frac{\csc \theta + \cot \theta}{\tan \theta + \sin \theta} = \cot \theta \csc \theta$
 - $\sec(\pi - x) = -\sec x$
 - $\frac{\sin(x + y)}{\cos x \cos y} = \tan x + \tan y$
 - $\sin 4\theta = 4 \sin \theta \cos \theta - 8 \sin^3 \theta \cos \theta$
 - $\frac{\tan(A + B) - \tan B}{1 + \tan(A + B) \tan B} = \tan A$
 - $\frac{\cot \alpha - \tan \alpha}{\cot \alpha + \tan \alpha} = \cos 2\alpha$
- Use sum and difference identities to find each exact values. (Do not use a calculator.)
 - $\cos \frac{\pi}{12}$
 - $\sin 105^\circ$
 - $\tan \frac{5\pi}{12}$
 - $\cos 75^\circ$
 - $\sin \frac{11\pi}{12}$
 - $\tan(165^\circ)$
- For each of the following, find $\sin(x - y)$, $\cos(x + y)$, $\tan(x - y)$, and the quadrant of $(x - y)$.
 - $\sin y = -\frac{2}{3}$, $\cos x = -\frac{1}{5}$, x in quadrant II, y in quadrant III
 - $\cos x = \frac{5}{13}$, $\cos y = \frac{24}{25}$, x in quadrant I, y in quadrant IV
 - $\sin x = -\frac{\sqrt{2}}{4}$, $\cos y = \frac{\sqrt{6}}{3}$, x in quadrant III, y in quadrant IV

6. Tell whether each statement is true or false.

a. $\cos(-24^\circ) = \cos 16^\circ - \cos 40^\circ$

b. $\cos\left(\theta - \frac{\pi}{2}\right) = \sin \theta$

c. $\tan(A + B) = \tan A + \tan B$

d. $\sin\left(\theta + \frac{\pi}{2}\right) = \sin \theta$

e. $\cos 85^\circ \cos 40^\circ + \sin 85^\circ \sin 40^\circ = \frac{\sqrt{2}}{2}$

7. Find each of the following.

a. $\sin A$, $\cos A$, given $\cos 2A = \frac{1}{8}$, A in quadrant IV

b. $\tan 2x$, given $\tan x = 5$

c. $\cos \frac{A}{2}$, given $\cos A = -\frac{3}{4}$, A in quadrant II

d. $\tan \frac{\theta}{2}$, given $\tan \theta = \frac{\sqrt{6}}{3}$, θ in quadrant III

e. $\sin \frac{B}{2}$, given $\sin B = -\frac{1}{5}$, B in quadrant IV

8. Write each expression as a single trigonometric function.

a. $\frac{\sin 2x}{1 + \cos 2x}$

d. $2 \cos^2 35^\circ - 1$

b. $\csc x - \cot x$

e. $2 \sin 5\theta \cos 5\theta$

c. $\sqrt{\frac{1 + \cos 80^\circ}{2}}$

f. $\frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$

9. Use half-angle identities to find each exact value. (Do not use a calculator.)

a. $\sin 22.5^\circ$

d. $\sin \frac{11\pi}{12}$

b. $\tan 75^\circ$

e. $\tan 67.5^\circ$

c. $\cos \frac{\pi}{12}$

f. $\cot \frac{5\pi}{8}$

10. Find the exact value of the real number y in the following. Do not use a calculator.

a. $y = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

d. $y = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

b. $y = \csc^{-1}(2)$

e. $y = \sec^{-1} \sqrt{2}$

c. $y = \cot^{-1} 0$

f. $y = \tan^{-1} \sqrt{3}$

11. Use calculator to find the decimal degree measure of θ , accurate to two decimal places.

a. $\theta = \cot^{-1}(-\sqrt{2})$

d. $\theta = \sec^{-1}(\sqrt{3})$

b. $\theta = \tan^{-1}(30)$

e. $\theta = \cos^{-1}(-.9645)$

c. $\theta = \sin^{-1}(.12345)$

f. $\theta = \csc^{-1}(-1.2308)$

12. Find each value without using a calculator.

a. $\sin\left(\cos^{-1}\frac{1}{4}\right)$

d. $\cos\left(\tan^{-1}\frac{5}{12} - \cot^{-1}\frac{4}{3}\right)$

b. $\cos\left(\tan^{-1}\frac{8}{3}\right)$

e. $\sin\left(2\cos^{-1}\frac{2}{9}\right)$

c. $\tan\left(2\sin^{-1}\frac{3}{5}\right)$

f. $\tan\left(\cos^{-1}\frac{3}{5} + \sin^{-1}\left(-\frac{5}{13}\right)\right)$

13. Solve each equation for x .

a. $y = 3\cot(2x - 1)$

d. $\tan^{-1}x = \cos^{-1}\frac{7}{25}$

b. $y = -3 + 2\sin 3(x + 5)$

e. $\sin^{-1}\frac{x}{2} + \sin^{-1}x = \frac{\pi}{2}$

c. $\cos^{-1}x - 2\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

14. Solve each equation for x in $[0, 2\pi)$ or θ in $[0^\circ, 360^\circ)$. Do not use calculator.

a. $\tan^3 x - 3\tan x = 0$

d. $\cot^2 3\theta - 1 = 0$

b. $\cos^2 \theta = \sin^2 \theta - 1$

e. $\sin 2x - \cos x = 0$

c. $2\sin^2 x + 5\sin x + 2 = 0$

f. $2\cos^2 2\theta = 1 - \cos 2\theta$

15. Write each of the following as an algebraic expression in x .

a. $\cos\left(\tan^{-1}\frac{x}{\sqrt{1-x^2}}\right)$

b. $\tan\left(\sec^{-1}\frac{\sqrt{1+x^2}}{x}\right)$

c. $\sin(2\cot^{-1}x)$